

Quiz 6

This quiz is graded out of 6 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §2.1 #41 Prove that the equation of the line through the distinct points (a_1, b_1) and (a_2, b_2) can be written as

$$\begin{vmatrix} x & y & 1 \\ a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \end{vmatrix} = 0 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} - \begin{vmatrix} x & y \\ a_2 & b_2 \end{vmatrix} + \begin{vmatrix} x & y \\ a_1 & b_1 \end{vmatrix}$$

$$0 = a_1 b_2 - a_2 b_1 - (x b_2 - y a_2) + (x b_1 - y a_1)$$

$$0 = a_1 b_2 - a_2 b_1 - x b_2 + y a_2 + x b_1 - y a_1$$

$$0 = a_1 b_2 - a_2 b_1 - x(b_2 - b_1) + y(a_2 - a_1)$$

$$(a_2 - a_1)y = (b_2 - b_1)x + a_2 b_1 - a_1 b_2$$

$$y = \frac{(b_2 - b_1)}{(a_2 - a_1)}x + \frac{a_2 b_1 - a_1 b_2}{(a_2 - a_1)}$$

$$y = mx + \frac{a_2 b_1 - a_1 b_2}{a_2 - a_1}$$

$$y - b_2 = mx + \frac{a_2 b_1 - a_1 b_2}{a_2 - a_1} - b_2$$

$$y - b_2 = mx + \frac{a_2 b_1 - a_1 b_2}{a_2 - a_1} - b_2 + m a_2 - m a_2$$

$$y - b_2 = m(x - a_2) + \frac{a_2 b_1 - a_1 b_2}{a_2 - a_1} - b_2 \frac{(a_2 - a_1)}{a_2 - a_1} + \frac{(b_2 - b_1)}{a_2 - a_1} a_2$$

$$y - b_2 = m(x - a_2)$$

Question 2. §2.2 TF Determine whether the statement is true or false, and justify your answer.

If the sum of the second and fourth row vectors of a 6×6 matrix A is equal to the last row vector, then $\det(A) = 0$.

$$|A| = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{16} \\ a_{21} & a_{22} & \dots & a_{26} \\ a_{31} & a_{32} & \dots & a_{36} \\ a_{41} & a_{42} & \dots & a_{46} \\ a_{51} & a_{52} & \dots & a_{56} \\ a_{21}+a_{41} & a_{22}+a_{42} & \dots & a_{26}+a_{46} \end{vmatrix} \xrightarrow{-R_2+R_6 \rightarrow R_6} \begin{vmatrix} a_{11} & a_{12} & \dots & a_{16} \\ a_{21} & a_{22} & \dots & a_{26} \\ a_{31} & a_{32} & \dots & a_{36} \\ a_{41} & a_{42} & \dots & a_{46} \\ a_{51} & a_{52} & \dots & a_{56} \\ a_{41} & a_{42} & \dots & a_{46} \end{vmatrix} \xrightarrow{-R_4+R_6 \rightarrow R_6} \begin{vmatrix} a_{11} & a_{12} & \dots & a_{16} \\ a_{21} & a_{22} & \dots & a_{26} \\ a_{31} & a_{32} & \dots & a_{36} \\ a_{41} & a_{42} & \dots & a_{46} \\ a_{51} & a_{52} & \dots & a_{56} \\ 0 & 0 & \dots & 0 \end{vmatrix}$$

$$= 0$$