

Quiz 7

This quiz is graded out of 8 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.7 #47

Prove: If $A^T A = A$, then A is symmetric and $A = A^2$.

premise:

$$\bullet A^T A = A$$

conclusion:

- A is symmetric
- $A = A^2$

Let's show that A is symmetric

$$A^T = (A^T A)^T \text{ by premise}$$

$$= A^T (A^T)^T$$

$$= A^T A$$

$$= A \text{ by premise}$$

Let's show that $A = A^2$

$$A = A^T A$$

$$= A A \text{ since } A \text{ is symmetric}$$

$$= A^2$$

Question 2. Problems for 201-NYC-05 Science #2.2

Let B be a 3×3 matrix where $\det(B) = 3$. Find $\det(2B + B^2 \text{adj}(B))$.

$$= \det(2B + B^2 (\det(B)) B^{-1})$$

$$= \det(2B + \det(B) B^2 B^{-1})$$

$$= \det(2B + 3 B B B^{-1})$$

$$= \det(2B + 3B)$$

$$= \det(5B)$$

$$= 5^3 \det(B)$$

$$= 5^3 (3)$$

$$\text{note: } B^{-1} = \frac{1}{\det(B)} \text{adj}(B)$$

$$(\det(B)) B^{-1} = \text{adj}(B)$$