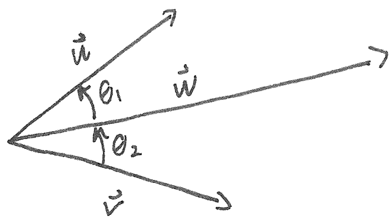
Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S2: Winter 2018

Name: Y. Lamontagne

Quiz 9

This quiz is graded out of 8 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §3.3 #38 Let $\vec{u}$ and $\vec{v}$ be nonzero vectors in 3-space, and let $k = ||\vec{u}||$ and $l = ||\vec{v}||$. Prove that the vector $\vec{w} = l\vec{u} + k\vec{v}$ bisects the angle between $\vec{u}$ and $\vec{v}$.



$$\vec{u}\cdot\vec{w} = ||\vec{u}||\,||\vec{w}||\cos\theta_1$$
$$\vec{u}\cdot(l\vec{u}+k\vec{v}) = ||\vec{u}||\,||\vec{w}||\cos\theta_1$$
$$\vec{u}\cdot(l\vec{u}) + \vec{u}\cdot(k\vec{v}) = ||\vec{u}||\,||\vec{w}||\cos\theta_1$$
$$l\vec{u}\cdot\vec{u} + k(\vec{u}\cdot\vec{v}) = k||\vec{w}||\cos\theta_1$$
$$l||\vec{u}||^2 + k(\vec{u}\cdot\vec{v}) = k||\vec{w}||\cos\theta_1$$
$$lk^2 + k(\vec{u}\cdot\vec{v}) = k||\vec{w}||\cos\theta_1$$
$$\frac{lk^2 + k(\vec{u}\cdot\vec{v})}{k||\vec{w}||} = \cos\theta_1$$
$$\frac{lk + \vec{u}\cdot\vec{v}}{||\vec{w}||} = \cos\theta_1$$

$$\vec{v}\cdot\vec{w} = ||\vec{v}||\,||\vec{w}||\cos\theta_2$$
$$\vec{v}\cdot(l\vec{u}+k\vec{v}) = ||\vec{v}||\,||\vec{w}||\cos\theta_2$$
$$\vec{v}\cdot(l\vec{u}) + \vec{v}\cdot(k\vec{v}) = ||\vec{v}||\,||\vec{w}||\cos\theta_2$$
$$l(\vec{v}\cdot\vec{u}) + k(\vec{v}\cdot\vec{v}) = ||\vec{v}||\,||\vec{w}||\cos\theta_2$$
$$l(\vec{u}\cdot\vec{v}) + k||\vec{v}||^2 = ||\vec{v}||\,||\vec{w}||\cos\theta_2$$
$$\frac{l(\vec{u}\cdot\vec{v}) + kl^2}{||\vec{v}||\,||\vec{w}||} = \cos\theta_2$$
$$\frac{\cancel{l}(\vec{u}\cdot\vec{v}) + kl^{\cancel{2}}}{\cancel{l}\,||\vec{w}||} = \cos\theta_2$$
$$\frac{\vec{u}\cdot\vec{v} + kl}{||\vec{w}||} = \cos\theta_2$$

$$\cos\theta_1 = \cos\theta_2$$
$$\arccos(\cos\theta_1) = \arccos(\cos\theta_2)$$
$$\theta_1 = \theta_2 \quad \text{since } 0 \le \theta_i < \pi$$

Question 2. §3.4 #TF determine whether the statement is true or false, and justify your answer. The general solution of the nonhomogeneous linear system $Ax = b$ can be obtained by adding b to the general solution of the homogeneous linear system $Ax = 0$.

False,

$$\underset{A}{\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix}}\underset{\vec{x}}{\begin{bmatrix}x\\ y\end{bmatrix}} = \underset{b}{\begin{bmatrix}1\\ 2\end{bmatrix}}$$

Solution of $Ax = 0$

$$\left[\begin{array}{cc|c}1 & 1 & 0\\ 2 & 2 & 0\end{array}\right] \sim_{-2R_1+R_2\to R_2} \left[\begin{array}{cc|c}1 & 1 & 0\\ 0 & 0 & 0\end{array}\right]$$

Let $y = t$ $\quad t \in \mathbb{R}$

$x + t = 0$

$x = -t$

$\therefore (x, y) = (-t, t)$

But $(x,y) = b + (-t,t) = (1,2) + (-t,t)$ is not the solution set of $Ax = b$ since $(1,2)$ is not a solution of $Ax = b$.

Question 3. §3.4 #TF determine whether the statement is true or false, and justify your answer. If $\mathbf{x}_1$ and $\mathbf{x}_2$ are two solutions of the nonhomogeneous linear system $A\mathbf{x} = \mathbf{b}$, then $\mathbf{x}_1 - \mathbf{x}_2$ is a solution of the corresponding homogeneous linear system.

True, if x_1 and x_2 are solutions of $Ax = b$ then $Ax_1 = b$ and $Ax_2 = b$.

It follows that $x_1 - x_2$ is a solution of $Ax = 0$ since
$$A(x_1 - x_2) = Ax_1 - Ax_2 = 0 - 0 = 0.$$