Dawson College: Winter 2019: Linear Algebra (SCIENCE): 201-NYC-05-S6: **Quiz 10** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Given the following planes: $\mathcal{P}_1 : x + y + kz = 0$, $\mathcal{P}_2 : x + ky + z = 0$, $\mathcal{P}_3 : kx + y + z = 0$. Find the values of k, if any, such that the intersection between the planes is the a point, a line, a plane.

$$\begin{bmatrix} 1 & 1 & K & 0 \\ 1 & K & 1 & 0 \\ K & 1 & 1 & 0 \end{bmatrix} \sim \begin{matrix} -R_1+R_2 \to R_2 \\ -KR_1+R_3 \to R_3 \end{matrix} \begin{bmatrix} 1 & 1 & K & 0 \\ 0 & K-1 & 1-K & 0 \\ 0 & 1-K & 1-K^2 & 0 \end{bmatrix} \sim_{R_2+R_3 \to R_3} \begin{bmatrix} 1 & 1 & K & 0 \\ 0 & K-1 & 1-K & 0 \\ 0 & 0 & 2-K-K^2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & K & 0 \\ 0 & K-1 & 1-K & 0 \\ 0 & 0 & -(K+2)(K-1) & 0 \end{bmatrix}$$

if $K \neq -2$, $K \neq 1$ then the system has a unique solution.

If $K = 1$ then $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Let $y = s$, $z = t$ $s,t \in \mathbb{R}$, $x = -s-t$

$\therefore (x,y,z) = (-s-t, s, t) = s(-1,1,0) + t(-1,0,1)$ $s,t \in \mathbb{R}$. $\therefore$ a plane

If $K = -2$ then $\begin{bmatrix} 1 & 1 & -2 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{matrix} \frac{1}{3}R_2+R_1 \to R_1 \\ -\frac{1}{3}R_2 \to R_2 \end{matrix} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Let $z = t$ $t \in \mathbb{R}$, $y = t$, $x = t$

$\therefore (x,y,z) = (t,t,t) = t(1,1,1)$ $t \in \mathbb{R}$ a line.

Question 2. Given two planes:

$$\begin{array}{lcrcrcrcr} \mathcal{P}_1 & : & x & - & 2y & + & z & = & 1 \\ \mathcal{P}_2 & : & -4x & + & 8y & - & 4z & = & -4 \end{array}$$

a. *(1 mark)* Give an argument to explain why the intersection of the two planes is a plane.

The two planes are multiple of each other. Hence the points that satisfy one plane satisfy the other

b. *(2 marks)* Find the solution set of the associated homogeneous linear system.

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ -4 & 8 & -4 & 0 \end{bmatrix} \sim_{4R_1+R_2 \to R_2} \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Let $y = s$, $z = t \Rightarrow x = 2s - t$ $s,t \in \mathbb{R}$

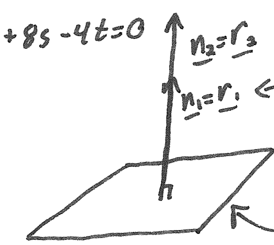
$(x,y,z) = (2s-t, s, t) = s(2,1,0) + t(-1,0,1)$

c. *(2 marks)* Show that the solution set of the associated homegeneous linear system is orthogonal to the rows of the coefficient matrix of the system.

$\underline{r}_1 \cdot \underline{x} = (1,-2,1) \cdot (2s-t, s, t) = 2s - t - 2s + t = 0$

$\underline{r}_2 \cdot \underline{x} = (-4,8,-4) \cdot (2s-t, s, t) = -8s + 4t + 8s - 4t = 0$

d. *(2 marks)* Give a geometrical interpretation to part c).



orthogonal to the normals. Hence to the rows of the coeff. matrix.

the solution set is a plane

e. *(1 mark)* Find a particular solution of the linear system by inspection.

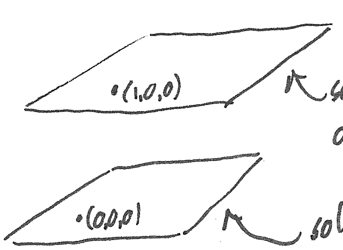
$(1,0,0)$

f. *(2 marks)* Without solving the linear system directly find the solution to the linear system.

$\underline{x}$ = particular sol. of $A\underline{x} = b$ + general sol. of $A\underline{x} = 0$ = $(1,0,0) + s(2,1,0) + t(-1,0,1)$

g. *(2 marks)* Give a geometrical interpretation on how the solution was found in part f).

sol. of $Ax = b$ is the sol. of $Ax = 0$ translated by $(1,0,0)$ a particular solution.



sol. of $Ax = 0$.