Dawson College: Winter 2019: Linear Algebra (SCIENCE): 201-NYC-05-S6: **Quiz 11** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(1 mark each)* Complete the following sentences with the word **must**, **might** or, **cannot**, as appropriate.

a. If $\vec{u}$ and $\vec{v}$ are nonzero vectors in $\mathbb{R}^3$, then $(\vec{u}\times\vec{v})\cdot\vec{u}$ must be equal to 0.

Question 2.[1] *(5 marks)* Let $\vec{u}$ and $\vec{v}$ be non-zero vectors in $\mathbb{R}^3$. Show that if $\frac{1}{\vec{u}\cdot\vec{v}}(\vec{u}\times\vec{v})$ is a unit vector then the angle between $\vec{u}$ and $\vec{v}$ is 45° or 135°.

$$\left\|\frac{1}{\underline{u}\cdot\underline{v}}(\underline{u}\times\underline{v})\right\| = 1$$

$$\frac{1}{|\underline{u}\cdot\underline{v}|}\|\underline{u}\times\underline{v}\| = 1$$

$$\|\underline{u}\times\underline{v}\| = |\underline{u}\cdot\underline{v}|$$

$$\|\underline{u}\|\|\underline{v}\|\sin\theta = \|\underline{u}\|\|\underline{v}\||\cos\theta|$$

$$\frac{\sin\theta}{|\cos\theta|} = 1$$

since $\theta\in[0,\pi)$

$$\left|\frac{\sin\theta}{\cos\theta}\right| = 1$$

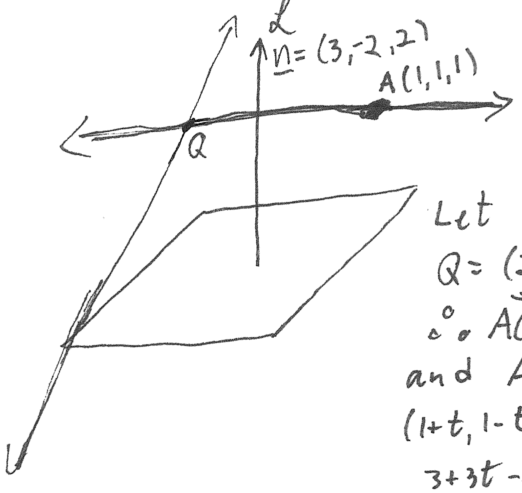
$$|\tan\theta| = 1$$

$\tan\theta = 1$ and $\tan\theta = -1$

$\theta = 45°$ $\quad\theta = 135^0$

Question 3.[2] Given the line $\mathcal{L}: (x,\ y,\ z) = (2,\ 2,\ 3) + t(1,\ -1,\ -3)$ where $t\in\mathbb{R}$, the plane $\mathcal{P}: 3x - 2y + 2z = 7$ and the point $A(1,\ 1,\ 1)$.

a. *(5 marks)* Find parametric equations of the line which contains A, intersects $\mathcal{L}$ and which is parallel to $\mathcal{P}$.



$\underline{X} = \underline{A} + t\vec{AQ}$ where $\vec{AQ}$ is parallel to $\mathcal{P}$, hence $\vec{AQ}\perp\vec{d}$ and such that the line has an intersection with $\mathcal{L}$.

Let Q be an arbitrary point on $\mathcal{L}$. That is $Q = (2+t,\ 2-t,\ 3-3t)$.

$\therefore\ \vec{AQ} = Q - A = (2+t, 2-t, 3-3t) - (1,1,1) = (1+t, 1-t, 2-3t)$

and $\vec{AQ}\cdot\underline{n} = 0$

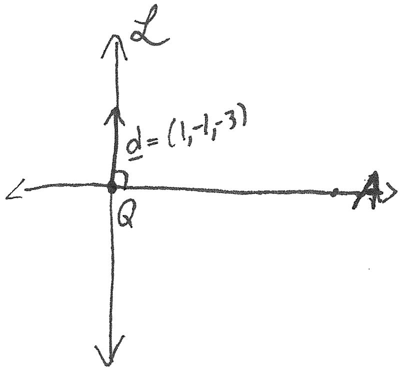
$$(1+t, 1-t, 2-3t)\cdot(3,-2,2) = 0$$

$$3+3t-2+2t+4-6t = 0$$

$$t = 5$$

$\therefore\ \underline{X} = (1,1,1) + t(1+5, 1-5, 2-3(5))$
$= (1,1,1) + t(6,-4,-13).$

b. *(5 marks)* Find parametric equations of the line which contains A and which intersects $\mathcal{L}$ at a right angle.



$\underline{X} = \underline{A} + t\vec{AQ}$ where $\vec{AQ}$ is a vector $\perp\mathcal{L}$ and such that the line has an intersection with $\mathcal{L}$.

Let Q be an arbitrary point on $\mathcal{L}$. That is $Q = (2+t, 2-t, 3-3t)$.

$\therefore\ \vec{AQ} = Q - A = (2+t, 2-t, 3-3t) - (1,1,1) = (1+t, 1-t, 2-3t)$

and

$$\vec{AQ}\cdot\underline{d} = 0$$

$$(1+t, 1-t, 2-3t)\cdot(1,-1,-3) = 0$$

$$1+t-1+t-6+9t = 0$$

$$11t = 6$$

$$t = \frac{6}{11}$$

$\therefore\ \underline{X} = (1,1,1) + t\left(1+\frac{6}{11}, 1-\frac{6}{11}, 2-3\left(\frac{6}{11}\right)\right)$
$= (1,1,1) + t\left(\frac{17}{11}, \frac{5}{11}, \frac{4}{11}\right)$

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[1] From John Abbott Final Examinations.

[2] From a Dawson College Final Examination.