Dawson College: Winter 2019: Linear Algebra (Science): 201-NYC-05-S6: **Quiz 12** name: ________________________________

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] Let $V = \mathbb{R}^2$. For $(u_1, u_2), (v_1, v_2) \in V$ and $a \in \mathbb{R}$ define vector addition by $(u_1, u_2) \boxplus (v_1, v_2) := (u_1 + v_1 + 2, u_2 + v_2 + 1)$ and scalar multiplication by $a \boxdot (u_1, u_2) := (au_1 + 2a - 2, au_2 + a - 1)$. It can be shown that $(V, \boxplus, \boxdot)$ is a vector space over the scalar field $\mathbb{R}$. Find the following:

a. *(1 mark)* $(3, 3) \boxplus (-9, -6)$

b. *(1 mark)* $-6 \boxdot (3, 3)$

c. *(2 marks)* $\underline{0}$

d. *(2 marks)* the additive inverse of (x, y)

Question 2. *(5 marks)* In any vector space V, for any $\vec{v} \in V$ and $a, b \in \mathbb{R}$ prove that if $a\vec{v} = b\vec{v}$ and $\vec{v} \neq 0$ then $a = b$. *Justify every step!!!*

Question 3. *(3 marks)* Prove that if the set of all points in $\mathbb{R}^3$ lying in a plane is a vector space with respect to the standard operations of vector addition and scalar multiplication then the plane passes through the origin.

Question 4. *(2 marks)* Determine whether the following is a vector space: $V = \{A | A \in \mathcal{M}_{2\times 2} \text{ and } A^T = A\}$ with the following operations:

$$A + B = AB \text{ and } kA = kA$$

That is, vector addition is matrix multiplication and scalar multiplication is the regular scalar multiplication. Justify.

[1] From WeBWorK.