Dawson College: Winter 2019: Linear Algebra (SCIENCE): 201-NYC-05-S6: **Quiz 12** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] Let $V = \mathbb{R}^2$. For $(u_1, u_2), (v_1, v_2) \in V$ and $a \in \mathbb{R}$ define vector addition by $(u_1, u_2) \boxplus (v_1, v_2) := (u_1+v_1+2, u_2+v_2+1)$ and scalar multiplication by $a \boxdot (u_1, u_2) := (au_1 + 2a - 2, au_2 + a - 1)$. It can be shown that $(V, \boxplus, \boxdot)$ is a vector space over the scalar field $\mathbb{R}$. Find the following:

a. *(1 mark)* $(3,3) \boxplus (-9,-6) = (3+(-9)+2, 3+(-6)+1) = (-4,-2)$

b. *(1 mark)* $-6 \boxdot (3,3) = ((-6)(3)+2(-6)-2, (-6)(3)+(-6)-1) = (-32, -25)$

c. *(2 marks)* $\underline{0}$ since $(V, \boxplus, \boxdot)$ is a vector space we have shown in class that

$\underline{0} = 0 \boxdot \underline{v}$

$\underline{0} = 0 \boxdot (x,y) = (0x+2(0)-2, 0(y)+0-1) = (-2,-1)$

d. *(2 marks)* the additive inverse of (x, y)

Since $(V, \boxplus, \boxdot)$ is a vector space we have shown in class that $(-1)\boxdot(x,y)$ is the additive inverse of (x,y)

$= ((-1)x+2(-1)-2, (-1)y+(-1)-1)$

$= (-x-4, -y-2)$

Question 2. *(5 marks)* In any vector space V, for any $\vec{v} \in V$ and $a, b \in \mathbb{R}$ prove that if $a\vec{v} = b\vec{v}$ and $\vec{v} \neq 0$ then $a = b$. *Justify every step!!!*

$a\underline{v} = b\underline{v}$

$a\underline{v} + ((-1)b\underline{v}) = b\underline{v} + ((-1)b\underline{v})$ add the additive inverse of $b\underline{v}$ on both sides. Note we have shown that if V is a vector space then the additive inverse of $\underline{v}$ is $(-1)\underline{v}$

$a\underline{v} + (-b)\underline{v} = b\underline{v} + (-b)\underline{v}$ by axiom 9

$(a-b)\underline{v} = (b-b)\underline{v}$ by axiom 8

$(a-b)\underline{v} = 0\underline{v}$

$(a-b)\underline{v} = \underline{0}$ by thm 1.1.1

then by thm 1.1.4 $a-b=0$ or $\underline{v}=\underline{0}$ but since $\underline{v} \neq \underline{0}$ then $a-b=0$

$a=b$.

Question 3. *(3 marks)* Show that if the set of all points in $\mathbb{R}^3$ lying in a plane is a vector space with respect to the standard operations of vector addition and scalar multiplication then the plane passes through the origin.

Let V be the set where $V = \{(x,y,z) \mid ax+by+cz=d\} \subset \mathbb{R}^3$

Suppose that the plane does not pass through the origin then $d \neq 0$. Let $\underline{u} = (x_1,y_1,z_1)$, $\underline{v} = (x_2,y_2,z_2) \in V$ then $\underline{u}+\underline{v} = (x_1+x_2, y_1+y_2, z_1+z_2) \notin V$ since

$a(x_1+x_2) + b(y_1+y_2) + c(z_1+z_2) = ax_1 + by_1 + cz_1 + ax_2 + by_2 + cz_2$

$= d + d$ since $\underline{u}, \underline{v} \in V$

$= 2d \neq d$ since $d=0$ (axiom 1 fails)

Contradicting the premise that V is a vector space. $\therefore d=0$

Question 4. *(2 marks)* Determine whether the following is a vector space: $V = \{A | A \in \mathcal{M}_{2\times2}$ and $A^T = A\}$ with the following operations:

$A + B = AB$ and $kA = kA$

That is, vector addition is matrix multiplication and scalar multiplication is the regular scalar multiplication. Justify.

Not a vector space since not closed under addition. $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \in V$

$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} +_V \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 3 & -3 \end{bmatrix} \notin V.$

[1] From WeBWorK.