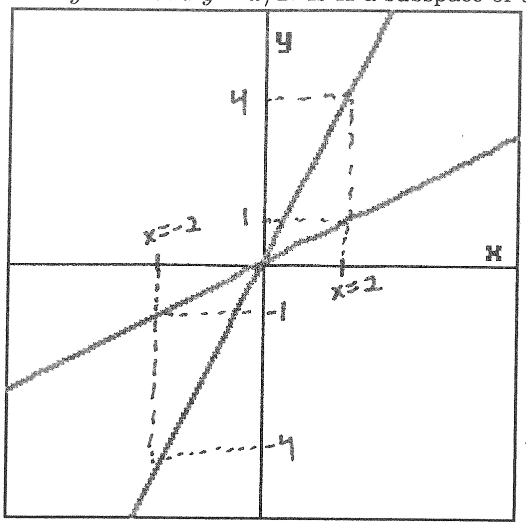
Dawson College: Winter 2019: Linear Algebra (Science): 201-NYC-05-S6: Quiz 13 name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Interpretation of between: strictly between the two lines

Question 1.[1] *(5 marks)* Let $V = \mathbb{R}^2$ and let H be the subset of V of all points in the first and third quadrants that lie between the lines $y = 2x$ and $y = x/2$. ~~Is H a subspace of the vector space V?~~ Determine whether H is closed under vector addition and scalar multiplication.



closed under addition? Let $(2,3.9), (-2,-1.1) \in H$

then $(2,3.9)+(-2,-1.1) = (0,2.8) \notin H$ since it does not lie between the lines $y=2x$ and $y=x/2$.

closed under scalar multiplication? let $(2,3.9) \in H$ and if $r=0$ then $0\cdot(2,3.9) = (0,0) \notin H$. since $(0,0)$ does not lie between the lines $y=2x$ and $y=\frac{x}{2}$

If the interpretation of between the two lines is between or on the line then H is closed under scalar multiplication. Let $(x,y) \in H$ then $\frac{x}{2} \leq y \leq 2x$ and it follows that for any r, $r(x,y) \in H$ because $r\frac{x}{2} < y < r2x$.

Question 2. *(5 marks)* Prove: If a nonempty subset W of a vector space V is closed under addition and scalar multiplication, under the inherited operations then W is a vector space.

Since W is closed under addition and scalar mult. axiom 1 and 6 hold

Since W is closed under scalar mult. and $\exists\, \underline{v} \in W$ the $0\underline{v} \in W$ and by thm 1.1 we have shown that $0\underline{v} = \underline{0}$. $\therefore$ $\underline{0} \in W$. axiom 4 holds

similarly $(-1)\cdot\underline{v} \in W$ and by thm 1.1 we have shown that $(-1)\cdot\underline{v}$ is the additive inverse of $\underline{v}$. $\therefore$ axiom 5 holds. Also axiom 2, 3, 7, 8, 9, and 10 hold. Since they hold for the superset V they must hold for the set W.

Question 3. *(2 marks)* $\mathbb{R}^3$ has infinitely many subspaces. Do every non-trivial space have infinitely many subspaces?

No, $\mathbb{R}$ only has itself or the trivial subspace. Suppose W is a subspace of V but not itself or $\{\underline{0}\}$. Then $\exists\, \underline{x} \in \mathbb{R}$ and $\underline{x} \notin W$ also $\exists\, \underline{y} \in W$ s.t. $\underline{y} \neq 0$. But $\exists\, r \in \mathbb{R}$ s.t. $r\underline{y} = \underline{x}$ and since W is a subspace $\underline{x} = r\underline{y} \in W$ $\lightning$

Question 4. *(2 marks)* Is $\mathbb{R}^2$ a subspace of $\mathbb{R}^3$?

No, since $\mathbb{R}^2$ is not a subset of $\mathbb{R}^3$

Question 5. *(2 marks)* If A is a subspace of a vector space V, is its complement $A^c = \{\vec{x} \in V \mid \vec{x} \notin A\}$ a subspace of V?

No, since A is a subspace $\underline{0} \in A$ but $\underline{0} \notin A^c$. $\therefore$ A^c does not contain the zero vector. $\therefore$ not a vector space.

[1] From WeBWorK.