Dawson College: Winter 2019: Linear Algebra (SCIENCE): 201-NYC-05-S6: **Quiz 14** name: ____

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(1 mark each)* Complete the following sentences with the word **must**, **might** or, **cannot**, as appropriate.

a. Given the identity: $\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$. If $\{\vec{u}, \vec{v}\}$ is linearly independent then $\{\vec{u}, \vec{v}, \vec{u} \times (\vec{v} \times \vec{w})\}$ ____________ be linearly independent.

b. Let $\vec{u}$ and $\vec{v}$ be distinct nonzero vectors in $\mathbb{R}^3$. If $\{\vec{u}, \vec{v}, \vec{w}\}$ is linearly independent, then $\vec{u} \cdot (\vec{v} \times \vec{w})$ ____________ be equal to $\vec{u} \cdot (\vec{w} \times \vec{v})$.

c. If $Ax = b$ has two distinct solutions then the columns of A ____________ be linearly dependent.

d. If $\{\vec{a}, 2\vec{a} + 3\vec{b}, \vec{a} - 3\vec{c}\}$ is linearly independent in a vector space V, then $\{\vec{a}, \vec{b}, \vec{c}\}$ ____________ be linearly independent.

e. If B has no column of zeros, but AB does, then the columns of A ____________ be linearly independent.

Question 2.[1] *(1 mark)* Fill in the correct numerical value. Suppose that $\{(3, -2, 7),\ (-2, a, b)\}$ is linearly dependent then $a =$ ______ and $b =$ _____

Question 3.[1] *(2 marks)* Write the definition of *linearly independent.* Be precise!

Question 4.[1] *(3 marks each)* Determine whether the following statements are true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. If $\{\vec{a}, \vec{b}\}$ and $\{\vec{u}, \vec{v}\}$ are both linearly dependent sets, then either $\{\vec{a}, \vec{u}\}$ or $\{\vec{a}, \vec{v}\}$ must be linearly dependent.

b. Suppose that $\{\vec{a}, \vec{b}\}$ and $\{\vec{u}, \vec{v}\}$ are both linearly independent sets. Then that either $\{\vec{a}, \vec{u}\}$ or $\{\vec{a}, \vec{v}\}$ must be linearly independent.

[1] From or modified from John Abbott College.