Dawson College: Winter 2019: Linear Algebra (SCIENCE): 201-NYC-05-S6: **Quiz 14** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(1 mark each)* Complete the following sentences with the word **must**, **might** or, **cannot**, as appropriate.

a. Given the identity: $\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$. If $\{\vec{u}, \vec{v}\}$ is linearly independent then $\{\vec{u}, \vec{v}, \vec{u} \times (\vec{v} \times \vec{w})\}$ might be linearly independent.

b. Let $\vec{u}$ and $\vec{v}$ be distinct nonzero vectors in $\mathbb{R}^3$. If $\{\vec{u}, \vec{v}, \vec{w}\}$ is linearly independent, then $\vec{u} \cdot (\vec{v} \times \vec{w})$ cannot be equal to $\vec{u} \cdot (\vec{w} \times \vec{v})$.

c. If $Ax = b$ has two distinct solutions then the columns of A must be linearly dependent.

d. If $\{\vec{a}, 2\vec{a} + 3\vec{b}, \vec{a} - 3\vec{c}\}$ ~~are~~ is linearly independent ~~vectors~~ in a vector space V, then $\{\vec{a}, \vec{b}, \vec{c}\}$ must be linearly independent.

~~e. If $\{\vec{a}, \vec{b}, \vec{c}\}$ is a linearly independent set in span($\{\vec{u}, \vec{v}, \vec{w}\}$), then $\{\vec{u}, \vec{v}, \vec{w}\}$ ________ be a linearly independent set.~~

f. If B has no column of zeros, but AB does, then the columns of A cannot be linearly independent.

Question 2.[1] *(1 mark)* Fill in the correct numerical value. Suppose that $\{(3, -2, 7),\ (-2, a, b)\}$ is linearly dependent then $a =$ $4/3$ and $b =$ $-14/3$

Question 3.[1] *(1 mark)* Write the definition of *linearly independent*. Be precise!

a nonempty set of vectors $\{V_1, V_2, \ldots, V_n\}$ is said to be linearly independent if

$$0 = c_1 V_1 + c_2 V_2 + \cdots + c_n V_n$$

only has the trivial solution, that is, $c_1 = c_2 = \cdots = c_n = 0$.

Question 4.[1] *(3 marks each)* Determine whether the following statements are true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. If $\{\vec{a}, \vec{b}\}$ and $\{\vec{u}, \vec{v}\}$ are both linearly dependent sets, then either $\{\vec{a}, \vec{u}\}$ or $\{\vec{a}, \vec{v}\}$ must be linearly dependent.

False,
Let $a = (1,0)$
$b = (2,0)$ then $\{a, b\}$ is lin. dep. since a is a multiple of b.

and
$u = (0,1)$
$v = (0,2)$ then $\{u, v\}$ is lin. dep. since u is a multiple of v.

But $\{a, u\}$ is lin. ind. since a is not a multiple of u.
$\{a, v\}$ is lin. ind. since a is not a multiple of v.

b. Suppose that $\{\vec{a}, \vec{b}\}$ and $\{\vec{u}, \vec{v}\}$ are both linearly independent sets. Then either $\{\vec{a}, \vec{u}\}$ or $\{\vec{a}, \vec{v}\}$ must be linearly independent.

True, Lets prove by contradiction. Suppose $\{a, u\}$ and $\{a, v\}$ are linearly dependent then $\exists k, l$ s.t. $a = ku$ and $a = lv$. It follows that

$$ku = lv$$
$$u = \frac{l}{k} v \quad k \neq 0 \text{ since by the premise } a \neq 0,\ b \neq 0,\ u \neq 0,\ v \neq 0$$

and therefore u is a multiple of v $\therefore$ $\{u, v\}$ is linearly dependent. ↯

$\therefore$ $\{a, u\}$ or $\{a, v\}$ is linearly independent.

[1]From or modified from John Abbott College.