Dawson College: Winter 2019: Linear Algebra (SCIENCE): 201-NYC-05-S6: **Quiz 15** name: ______________________

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Definitions. Let A be an $m \times n$ matrix.
The set of all $\mathbf{x} \in \mathbb{R}^n$ such that $A\mathbf{x} = \mathbf{0}$ is called the *null space* of A, that is, $\mathrm{Nul}(A) = \{\mathbf{x} \mid A\mathbf{x} = \mathbf{0}\}$.
The set of all $\mathbf{x} \in \mathbb{R}^m$ such that $\mathbf{x}$ is in the span of the vectors that are the columns of A is called the *column space* of A, that is, if $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n$ are the columns of A then $\mathrm{Col}(A) = \mathrm{span}(\{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n\})$.

Question 1.[1] *(1 mark each)* Complete the following sentences with the word **must**, **might** or, **cannot**, as appropriate.

a. The columns of an $n \times n$ elementary matrix ____________ be a basis for $\mathbb{R}^n$.

b. If A is an $m \times n$ matrix and $\dim(\mathrm{Nul}(A)) = n$, then A ____________ be a $m \times n$ zero matrix

Question 2. *(1 mark each)* Fill in the blank.

a. The vector space of all diagonal $n \times n$ matrices has dimension ______.

b. The vector space of all skew-symmetric (i.e. $A^T = -A$) $n \times n$ matrices has dimension ______.

Question 3.[1] Consider the following matrix A and its reduced row echelon form B given below. Justify completely!

$$A = \begin{bmatrix} 2 & 1 & -1 & 1 & 0 & 3 \\ 3 & 4 & -14 & 1 & 0 & 14 \\ 1 & -1 & 7 & 2 & 0 & -9 \\ -3 & -2 & 4 & 3 & 0 & -24 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 & 2 \\ 0 & 1 & -5 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

a. *(4 marks)* Find a basis for $\mathrm{Col}(A)$.

b. *(4 marks)* Find a basis for $\mathrm{Nul}(A)$.

c. *(1 mark)* Express the third column of matrix A relative to the basis found in part a.

d. *(1 mark)* Determine the dimension of $\mathrm{Col}(A)$ and $\mathrm{Nul}(A)$.

[1] From or modified from John Abbott College Final Examination.