Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S6: Winter 2019

Name: Y. Lamontagne

Quiz 2

This quiz is graded out of 12 marks. No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(6 marks)* Given that (0, 0, 0, 0) is a particular solution of a system of linear equations with coefficient matrix A where

$$A = \begin{bmatrix} 2 & 1 & 0 & -1 \\ 3 & 3 & 2 & -1 \\ 5 & 4 & 2 & -2 \end{bmatrix}, \quad \begin{aligned} 2x_1 + x_2 \quad\; - x_4 &= b_1 \\ 3x_1 + 3x_2 + 2x_3 - x_4 &= b_2 \\ 5x_1 + x_2 + 2x_3 - 2x_4 &= b_3 \end{aligned} \Rightarrow \begin{aligned} b_1 &= 2(0) + 0 - 0 = 0 \\ b_2 &= 3(0) + 3(0) + 2(0) - 0 = 0 \\ b_3 &= 5(0) + 0 + 2(0) - 2(0) = 0 \end{aligned}$$

then

find the augmented matrix of the system of linear equations and find the solution set using Gauss Jordan elimination.

$$\begin{bmatrix} 2 & 1 & 0 & -1 & 0 \\ 3 & 3 & 2 & -1 & 0 \\ 5 & 4 & 2 & -2 & 0 \end{bmatrix}$$

$$\sim \begin{matrix} 2R_2 \to R_2 \\ 2R_3 \to R_3 \end{matrix} \begin{bmatrix} 2 & 1 & 0 & -1 & 0 \\ 6 & 6 & 4 & -2 & 0 \\ 10 & 8 & 4 & -4 & 0 \end{bmatrix}$$

$$\sim \begin{matrix} -3R_1 + R_2 \to R_2 \\ -5R_1 + R_3 \to R_3 \end{matrix} \begin{bmatrix} 2 & 1 & 0 & -1 & 0 \\ 0 & 3 & 4 & 1 & 0 \\ 0 & +3 & 4 & 1 & 0 \end{bmatrix}$$

$$\sim \begin{matrix} \\ \\ -R_2 + R_3 \to R_3 \end{matrix} \begin{bmatrix} 2 & 1 & 0 & -1 & 0 \\ 0 & 3 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \; 3R_1 \to R_1 \begin{bmatrix} 6 & 3 & 0 & -3 & 0 \\ 0 & 3 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \; -R_2 + R_1 \to R_1 \begin{bmatrix} 6 & 0 & -4 & -4 & 0 \\ 0 & 3 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{matrix} \frac{1}{6}R_1 \to R_1 \\ \frac{1}{3}R_2 \to R_2 \end{matrix} \begin{bmatrix} 1 & 0 & -2/3 & -2/3 & 0 \\ 0 & 1 & 4/3 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Let $x_3 = s$, $x_4 = t$, $s, t \in \mathbb{R}$ $\Rightarrow$ $x_1 = \frac{2}{3}s + \frac{2}{3}t$, $x_2 = -\frac{4}{3}s - \frac{1}{3}t$

$\therefore (x_1, x_2, x_3, x_4) = \left(\frac{2}{3}s + \frac{2}{3}t, -\frac{4}{3}s - \frac{1}{3}t, s, t\right)$, $s, t \in \mathbb{R}$

Question 2.[1] *(6 marks)* Consider the system

$$\begin{aligned} kx &+ y + kz = 1 \\ x &+ y + z = 1 \\ (2-k)x &+ (2-k)y + z = 1 \\ kx &+ y + kz = k^2 \end{aligned}$$

$$\begin{bmatrix} k & 1 & k & 1 \\ 1 & 1 & 1 & 1 \\ 2-k & 2-k & 1 & 1 \\ k & 1 & k & k^2 \end{bmatrix} \sim R_1 \leftrightarrow R_2 \begin{bmatrix} 1 & 1 & 1 & 1 \\ k & 1 & k & 1 \\ 2-k & 2-k & 1 & 1 \\ k & 1 & k & k^2 \end{bmatrix}$$

Find the value(s) of k, if any, such that the system has: no solutions, a unique solution, infiniely many solutions.

$$\sim \begin{matrix} -kR_1 + R_2 \to R_2 \\ -(2-k)R_1 + R_3 \to R_3 \\ -kR_1 + R_4 \to R_4 \end{matrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1-k & 0 & 1-k \\ 0 & 0 & k-1 & k-1 \\ 0 & 1-k & 0 & k^2-k \end{bmatrix} \sim \; -R_2 + R_4 \to R_4 \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1-k & 0 & 1-k \\ 0 & 0 & k-1 & k-1 \\ 0 & 0 & 0 & k^2-1 \end{bmatrix}$$

no solutions: if $k^2 - 1 \neq 0$ $k \neq \pm 1$ then there is a leading entry in the constant column, hence no solutions

If $k = 1$ then $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and infinitely many solutions since # leading 1 < # var.

If $k = -1$ then $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and unique solution since # leading entries = # var.

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[1] From the Winter 2018 Dawson College Final Examination.