Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S6: Winter 2019

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Quiz 3

This quiz is graded out of 13 marks. No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(3 marks)* Find all matrices A, if any, such that

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A must be a 2×3 matrix.

$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} a+d & b+e & c+f \\ 2a+2d & 2b+2e & 2c+2f \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$a+d=0$
$b+e=0$
$c+f=0$
$2a+2d=0$
$2b+2e=0$
$2c+2f=0$

Let $d=t$, $e=s$, $f=r$ where $t,s,r \in \mathbb{R}$

so $a=-t$, $b=-s$, $c=-r$

$\therefore A = \begin{bmatrix} -t & -s & -r \\ t & s & r \end{bmatrix}$ $t,s,r \in \mathbb{R}$

Question 2.[1] *(4 marks)* Solve for X when:

$$\begin{bmatrix} -13 & 1 \\ 27 & 2 \end{bmatrix} X = \begin{bmatrix} -11 & \frac{1}{2} \\ 29 & 1 \end{bmatrix} X - \begin{bmatrix} -1 & 0 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} -11 & \frac{1}{2} \\ 29 & 1 \end{bmatrix} X - \begin{bmatrix} -13 & 1 \\ 27 & 2 \end{bmatrix} X$$

$$\begin{bmatrix} -1 & 0 \\ 2 & 2 \end{bmatrix} = \left(\begin{bmatrix} -11 & \frac{1}{2} \\ 29 & 1 \end{bmatrix} - \begin{bmatrix} -13 & 1 \\ 27 & 2 \end{bmatrix}\right) X$$

$$\begin{bmatrix} -1 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -\frac{1}{2} \\ 2 & -1 \end{bmatrix} X$$

$$\begin{bmatrix} 2 & -\frac{1}{2} \\ 2 & -1 \end{bmatrix}^{-1}\begin{bmatrix} -1 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -\frac{1}{2} \\ 2 & -1 \end{bmatrix}^{-1}\begin{bmatrix} 2 & -\frac{3}{2} \\ 2 & -1 \end{bmatrix} X$$

$$\frac{1}{-1}\begin{bmatrix} -1 & \frac{1}{2} \\ -2 & 2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 2 & 2 \end{bmatrix} = X$$

$$X = -\begin{bmatrix} 2 & 1 \\ 6 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ -6 & -4 \end{bmatrix}$$

Question 3. *(2 marks)* Determine whether the statement is true or false, and justify your answer.
The sum of two invertible matrices of the same size must be invertible.

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, both are invertible since in both cases $ad-bc \neq 0$. But $A+B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is not invertible since $ad-bc=0$.

Question 4. *(4 marks)* Show that if R is a $1 \times n$ matrix and C is an $n \times 1$ matrix, then $RC = \text{tr}(CR)$.

Let $R = [r_1 \ r_2 \ \cdots \ r_n]$ and $C = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$

$$RC = [r_1 \ r_2 \ \cdots \ r_n]\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = [r_1c_1 + r_2c_2 + \cdots + r_nc_n]$$
$$= r_1c_1 + r_2c_2 + \cdots + r_nc_n$$

$\text{tr}(CR)$

$$= \text{tr}\left(\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}[r_1 \ r_2 \ \cdots \ r_n]\right)$$

$$= \text{tr}\left(\begin{bmatrix} r_1c_1 & r_2c_1 & \cdots & r_nc_1 \\ r_1c_2 & r_2c_2 & \cdots & r_nc_2 \\ \vdots & \vdots & \ddots & \vdots \\ r_1c_n & r_2c_n & \cdots & r_nc_n \end{bmatrix}\right)$$

$$= r_1c_1 + r_2c_2 + \cdots + r_nc_n$$

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[1]From the Fall 2017 Dawson College Final Examination.