Question 1.¹ (5 marks) Prove: Any matrix A can be expressed as A = ER where E is an invertible matrix and R is a matrix in reduced row-echelon form.

Question 2. (4 marks) Find all values of c, if any, for which the given matrix is invertible.

 $\begin{bmatrix} c & c & c \\ 1 & c & c \\ 1 & 1 & c \end{bmatrix}$

Question 3. Determine whether the following statements are true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. (2 marks) An expression of an invertible matrix A as a product of elementary matrices is unique.

b. (2 marks) If A is invertible and a multiple of the first row of A is added to the second row, then the resulting matrix is invertible.

c. (2 marks) The product of two elementary matrices of the same size must be an elementary matrix.

 $^{^1\}mathrm{From}$ the Fall 2015 Dawson College Final Examination.