Dawson College: Winter 2019: Linear Algebra (SCIENCE): 201-NYC-05-S6: **Quiz 4** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(5 marks)* Prove: Any matrix A can be expressed as $A = ER$ where E is an invertible matrix and R is a matrix in reduced row-echelon form. If we perform Gauss-Jordan on $A \sim K$ elem. row op $\sim R$. For each elem. row op. we obtain an elem. mat. E_i. It follows that

$$E_K \cdots E_2 E_1 A = R$$

$$(E_K \cdots E_2 E_1)^{-1} E_K \cdots E_2 E_1 A = (E_K \cdots E_2 E_1)^{-1} R$$

since the product of inv. mat. is inv. and elem. matrices are inv.

$$I A = E_1^{-1} E_2^{-1} \cdots E_K^{-1} R$$

$$A = \underbrace{E_1^{-1} E_2^{-1} \cdots E_K^{-1}}_{E} R$$

E is inv. since it is a product of elem. mat.

Question 2. *(4 marks)* Find all values of c, if any, for which the given matrix is invertible.

$$\begin{bmatrix} c & c & c \\ 1 & c & c \\ 1 & 1 & c \end{bmatrix}$$

If $c = 0$.

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

not inv. by the equivalence thm. since the RREF is not I

Suppose $c \neq 0$

$$\begin{bmatrix} c & c & c \\ 1 & c & c \\ 1 & 1 & c \end{bmatrix} \sim_{\frac{1}{c}R_1 \to R_1} \begin{bmatrix} 1 & 1 & 1 \\ 1 & c & c \\ 1 & 1 & c \end{bmatrix} \sim_{\substack{-R_1+R_2 \to R_2 \\ -R_1+R_3 \to R_3}} \begin{bmatrix} 1 & 1 & 1 \\ 0 & c-1 & c-1 \\ 0 & 0 & c-1 \end{bmatrix}$$

The RREF is I if $c \neq 1$, since the matrix has three leading entries when $c \neq 1$.

$\therefore$ the matrix is invertible if $c \neq 0, 1$.

Question 3. Determine whether the following statements are true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. *(2 marks)* An expression of an invertible matrix A as a product of elementary matrices is unique.

False, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ← the identity expressed as the product of one elem. matrix

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ " " " " " " two " "

b. *(2 marks)* If A is invertible and a multiple of the first row of A is added to the second row, then the resulting matrix is invertible.

True, since A is invertable we have by the equivalence thm. $A \sim K$ elem. row op. $\sim I$. If $A \sim_{kR_1+R_2 \to R_2} B$ we obtain the resulting matrix. Then it follows that $B \sim_{-kR_1+R_2 \to R_2} A \sim K$ elem. row op from $A \sim I$. Hence B is invertible by the equivalence thm. since the RREF of B is I.

c. *(2 marks)* The product of two elementary matrices of the same size must be an elementary matrix.

False,

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$$

elem. mat

not an elem. mat.

[1] From the Fall 2015 Dawson College Final Examination.