Dawson College: Winter 2019: Linear Algebra (SCIENCE): 201-NYC-05-S6: **Quiz 5** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Definition. The number of non-zero rows of the row echelon form of a matrix is called the *rank of the matrix.*

Question 1.[1] *(3 marks)* Complete the following sentences with the word **must**, **might** or, **cannot**, as appropriate.

a. If the 3×3 coefficient matrix of the system $A\mathbf{x} = \mathbf{b}$ has a rank of 2, then the system might be inconsistent when $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

b. Given an $n \times n$ matrix A. If the system $A\mathbf{x} = \mathbf{b}$ is inconsistent for some $\mathbf{b} \in \mathbb{R}^n$, then the system $A\mathbf{x} = \mathbf{0}$ must have non-trivial solutions.

c. For any invertible matrix A, the rank of A must be the same as the rank of A^2.

Question 3 *(3 marks)* Given the following system $\begin{cases} x & = a \\ 2x + y & = b \\ z & = c \end{cases}$.

a. Find the inverse of the coefficient matrix.

$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b. For which value(s), if any, of a, b, c is the system consistent, justify.

Consistent for all a, b, c by the equivalence theorem since the coefficient matrix is invertible

c. Solve the system using the inverse for the value(s), if any, found in part b.

$A\underline{x} = \underline{b}$

$A^{-1}A\underline{x} = A^{-1}\underline{b}$

$I\underline{x} = A^{-1}\underline{b}$

$\underline{x} = A^{-1}\underline{b}$

$\underline{x} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ -2a+b \\ c \end{bmatrix}$

Question 2.[1] *(4 marks)* A matrix A is said to be *skew-symmetric* if $A^T = -A$. Given that $n \times n$ matrices A and B are skew-symmetric and $AB = -BA$, show that matrix AB must also be skew-symmetric.

Premise:
- $A^T = -A$
- $B^T = -B$
- $AB = -BA$

Conclusion:

AB is skew-symmetric

Need to show: $(AB)^T = -AB$

$LHS = (AB)^T$
$= B^T A^T$
$= -B(-A)$ from premise
$= BA$
$= -AB$ from premise

Question 3. Determine whether the following statements are true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. *(3 marks)* If B is a square matrix satisfying $AB = I$, then $B = A^{-1}$.

True, Consider $Bx = 0 \Rightarrow ABx = A0 \Rightarrow Ix = 0 \Rightarrow x = 0$ only trivial solution so by equivalence thm B is invertible. So $AB = I \Rightarrow ABB^{-1} = IB^{-1} \Rightarrow AI = B^{-1} \Rightarrow A = B^{-1} \Rightarrow A^{-1} = (B^{-1})^{-1} \Rightarrow A^{-1} = B$

b. *(2 marks)* If A^2 is a symmetric matrix, then A is a symmetric matrix.

False,

Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $A^2 = 0$ which is symmetric but A is not.

[1] From John Abbott Final Examinations.