Dawson College: Winter 2019: Linear Algebra (SCIENCE): 201-NYC-05-S6: **Quiz 7** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(3 marks)* Complete the following sentences with the word **must**, **might** or, **cannot**, as appropriate.

a. If A is a product of elementary matrices, then $det(A)$ cannot equal zero.

Let A and B be invertible $n \times n$ matrices. Let C be a non-invertible $n \times n$ matrix.

b. $A + C$ might be invertible.

c. AC and BC must have the same determinant.

Question 2.[2] *(5 marks)* Given A, an $n \times n$ matrix such that $\det(A) = 9$ and

$$A^3 A^T = 3A^{-1}\text{adj}(A)$$

find n.

$$\det(A^3A^T) = \det(3A^{-1}\text{adj}(A))$$
$$\det(A^3)\det(A^T) = 3^n \det(A^{-1}\text{adj}(A))$$
$$(\det A)^3 \det(A) = 3^n \det(A^{-1})\det(\text{adj}(A))$$
$$(\det(A))^4 = 3^n \frac{1}{\det(A)} (\det(A))^{n-1}$$
$$9^4 = 3^n \frac{1}{9}(9)^{n-1}$$
$$9^5 = 3^n (3^2)^{n-1}$$

$$(3^2)^5 = 3^n 3^{2n-2}$$
$$3^{10} = 3^{n+2n-2}$$
$$10 = n + 2n - 2$$
$$12 = 3n$$
$$4 = n$$

Question 3.[1] *(5 marks)* Let $\det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right)$ be a nonzero value n. Use Cramer's Rule to solve for x_3 only in the system of linear equations below:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & a & b & c \\ 0 & d & e & f \\ 0 & g & h & i \end{bmatrix}}_{A} \underset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}} = \underset{b}{\begin{bmatrix} 0 \\ 3b+2c \\ 3e+2f \\ 3h+2i \end{bmatrix}}$$

$$|A| = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & a & b & c \\ 0 & d & e & f \\ 0 & g & h & i \end{vmatrix} = a_{11}C_{11} = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = n$$

$$|A_3| = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 3b+2c & c \\ 0 & d & 3e+2f & f \\ 0 & g & 3h+2i & i \end{vmatrix} \underset{-2C_4 + C_3 \to C_3}{=} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 3b & c \\ 0 & d & 3e & f \\ 0 & g & 3h & i \end{vmatrix} \underset{\frac{1}{3}C_3 \to C_3}{=} 3\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & a & b & c \\ 0 & d & e & f \\ 0 & g & h & i \end{vmatrix} = 3n$$

$$X_3 = \frac{|A_3|}{|A|} = \frac{3n}{n} = 3$$

Question 4. *(2 marks)* Determine whether the following statements are true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.
If A and B are square matrices of the same size such that $\det(A) = \det(B)$, then $\det(A + B) = 2\det(A)$.

False Let $A = I_2$, $B = -I_2$, $\det(A) = 1 = \det(B)$ but $\det(A+B) = \det(O) = 0$
$\neq 2\det(A) = 2$

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[1]From John Abbott Final Examinations.

[2]From a Dawson College Final Examination.