Dawson College: Winter 2019: Linear Algebra (SCIENCE): 201-NYC-05-S6: **Quiz 5** name: ______________________

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Definition. The number of non-zero rows of the row echelon form of a matrix is called the *rank of the matrix.*

Question 1.[1] *(3 marks)* Complete the following sentences with the word **must**, **might** or, **cannot**, as appropriate.

a. If the 3×3 coefficient matrix of the system $A\mathbf{x} = \mathbf{b}$ has a rank of 2, then the system ____________ be inconsistent when $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

b. Given an $n \times n$ matrix A. If the system $A\mathbf{x} = \mathbf{b}$ is inconsistent for some $\mathbf{b} \in \mathbb{R}^n$, then the system $A\mathbf{x} = \mathbf{0}$ ____________ have non-trivial solutions.

c. For any invertible matrix A, the rank of A ____________ be the same as the rank of A^2.

Question 2. *(3 marks)* Given the following system $\begin{cases} x & = a \\ 2x + y & = b \\ z & = c \end{cases}$.

a. Find the inverse of the coefficient matrix.

b. For which value(s), if any, of a, b, c is the system consistent, justify.

c. Solve the system using the inverse for the value(s), if any, found in part b.

Question 3.[1] *(4 marks)* A matrix A is said to be *skew-symmetric* if $A^T = -A$. Given that $n \times n$ matrices A and B are skew-symmetric and $AB = -BA$, show that matrix AB must also be skew-symmetric.

Question 4. Determine whether the following statements are true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. *(3 marks)* If B is a square matrix satisfying $AB = I$, then $B = A^{-1}$.

b. *(2 marks)* If A^2 is a symmetric matrix, then A is a symmetric matrix.

[1]From John Abbott Final Examinations.