Supplementary Problems for 201-NYC-05

An open source collection of
48 CEGEP level linear algebra problems from 17 authors.

January 9, 2023

1 Matrix Inverses

1.1 [MH] Find the matrix X such that

$$(X^{\mathsf{T}} - 3I)\begin{bmatrix}1 & 3\\2 & 5\end{bmatrix} = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$$

1.2 [YL] Given $C = \begin{bmatrix}1 & 2\\5 & 0\end{bmatrix}$ and $D = \begin{bmatrix}-1 & 1\\4 & -3\end{bmatrix}$. Solve the given equations for X.

a. $CXD = 10I$

b. $C((DX)^{\mathsf{T}} - 2I)^{-1} = C$

1.3 [YL] Solve for X given that it satisfies

$$DXD^T = \mathrm{tr}(BC)BC$$

where

$$B = \begin{bmatrix}2 & 1 & 0\\-3 & 4 & 0\end{bmatrix}\ C = \begin{bmatrix}2 & -1\\3 & -2\\1 & 0\end{bmatrix}\ D = \begin{bmatrix}2 & -2\\1 & -2\end{bmatrix}.$$

1.4 [YL] Solve for X given that it satisfies

$$\left(2A + X^T\right)^{-1} = I$$

where

$$A = \begin{bmatrix}1 & 3\\1 & 2\end{bmatrix}$$

1.5 [YL] Solve for A given that it satisfies

$$(I - A^T)^{-1} = (\mathrm{tr}(B)B^2)^{\mathsf{T}}$$

where

$$B = \begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$$

1.6 [AG] Solve for A the following equation:

$$(3A^T - I)^{-1}C - D = 0$$

where $C = \begin{pmatrix}7 & -1\\6 & -1\end{pmatrix}$ and $D = \begin{pmatrix}5 & 4\\1 & 1\end{pmatrix}$.

1.7 [YL] Solve for X given that it satisfies

$$\left(\mathrm{tr}(A)A + X^{\mathsf{T}}A\right)^{-1} = \frac{1}{6}I_3$$

where

$$A = \begin{bmatrix}1 & 2 & 3\\0 & 2 & 3\\0 & 0 & 3\end{bmatrix}$$

1.8 [BS] Solve for A, if possible:

$$A^{-1}\begin{bmatrix}1 & 0\\-2 & -1\end{bmatrix} = \left(\begin{bmatrix}3 & 0\\5 & 3\end{bmatrix} - 2A\right)^{-1}$$

1.9 [BS] Find all matrices A such that

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\7 & 0 & 1\end{bmatrix} = (3I_3 - 2A)^{-1}$$

2 Properties of Determinants, Adjoint of a Matrix and Cramer's Rule

2.1 [MB] Given that the cofactor matrix of A is

$$\mathrm{cof}(A) = \begin{bmatrix}3 & -6 & 5\\-4 & 3 & -5\\5 & -5 & 5\end{bmatrix}$$

and that the first row of A is $\begin{bmatrix}2 & 1 & -1\end{bmatrix}$

a. Find $\mathrm{adj}(A)$.

b. Find $\det(A)$.

c. Find the minors M_{12} and M_{22} of A.

d. Find A^{-1}.

2.2 [YL] Given

$$A = \begin{bmatrix}0 & 1 & 2 & 0\\3 & 0 & 0 & 4\\5 & 0 & 0 & 6\\0 & 7 & 8 & 0\end{bmatrix}$$

.

a. Evaluate $\det(A)$.

b. Evaluate $\det\left(\mathrm{adj}\left((3A^{-1})^T\right)\right)$.

2.3 [YL] Let B be a 3×3 matrix where $\det(B) = 3$. Find

$$\det\left(2B + B^2\mathrm{adj}(B)\right).$$

2.4 [MB] Consider two 4×4 matrices A and B, with $\det(A) = -2$ and $\det(B) = 3$. Find the determinant of M, knowing that $\det(2B^{\mathsf{T}}MA^{-1}B) = \det(\mathrm{adj}(A)A^2B)$.

2.5 [AG] Let A and B be two 3×3 matrices with $\det(A) = 2$ and $\det(B) = 5$. Find $\det\left(A^3\det(B)A^{\mathsf{T}}(\mathrm{adj}(A))^4\right)$.

2.6 [**YL**] Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -2 & -3 & -4 \\ 2 & 3 & 0 & 0 \\ 0 & 4 & 5 & 0 \end{bmatrix}.$$

a. Evaluate $\det(A)$.

b. Given M is a 4×4 matrix such that $\det(M) = 2$, evaluate $\det(\det(M)\operatorname{adj}(5M^T A^{-1}))$.

c. Given M is a 4×4 matrix such that $\det(M) = 3$, evaluate $\det(\det(5A^T)\operatorname{adj}(MA^{-1}))$.

2.7 [**BS**] If A is a 3×3 matrix such that $\det(A) = 2$ find

$$\det\left(\operatorname{adj}\left(2\operatorname{adj}\left(A^{-1}\right)\right)\right)$$

.

2.8 [**BS**] Given A, an $n\times n$ matrix such that $\det(A) = 9$ and

$$A^3 A^T = 3A^{-1}\operatorname{adj}(A)$$

find n.

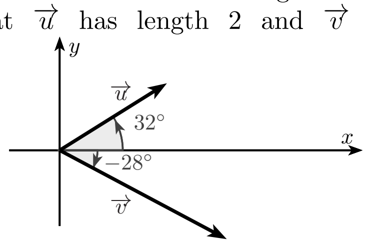
2.9 [**BS**] If A and B are invertible matrices of the same size show that

$$\operatorname{adj}(AB) = \operatorname{adj}(B)\operatorname{adj}(A)$$

.

3 Dot Product and Projections

3.1 [**MH**] Suppose that $\vec{u}$ and $\vec{v}$ are two vectors in the xy-plane with directions as given in the diagram and such that $\vec{u}$ has length 2 and $\vec{v}$ has length 3.



a. Find $\boldsymbol{u}\cdot\boldsymbol{v}$.

b. Find $\|\boldsymbol{u}+\boldsymbol{v}\|$.

3.2 [**BS**] If $\boldsymbol{u} = (0, 1, 1)$ and $\boldsymbol{v} = (p, 4, p)$ then find the parameter p such that the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is $\pi/3$.

3.3 [**SM**] Prove the parallelogram law for the norm:

$$\|\boldsymbol{a}+\boldsymbol{b}\|^2 + \|\boldsymbol{a}-\boldsymbol{b}\|^2 = 2\|\boldsymbol{a}\|^2 + 2\|\boldsymbol{b}\|^2$$

for all vectors in $\mathbb{R}^n$.

3.4 [**JH**] Show that $\|\boldsymbol{u}\| = \|\boldsymbol{v}\|$ if and only if $\boldsymbol{u}+\boldsymbol{v}$ and $\boldsymbol{u}-\boldsymbol{v}$ are perpendicular. Give an example in $\mathbb{R}^2$.

4 Lines

4.1 [**GHC**] Write the vector, parametric and symmetric equations of the lines described.

a. Passes through $P = (2, -4, 1)$, parallel to $\boldsymbol{d} = (9, 2, 5)$.

b. Passes through $P = (6, 1, 7)$, parallel to $\boldsymbol{d} = (-3, 2, 5)$.

c. Passes through $P = (2, 1, 5)$ and $Q = (7, -2, 4)$.

d. Passes through $P = (1, -2, 3)$ and $Q = (5, 5, 5)$.

e. Passes through $P = (0, 1, 2)$ and orthogonal to both $\boldsymbol{d}_1 = (2, -1, 7)$ and $\boldsymbol{d}_2 = (7, 1, 3)$.

f. Passes through $P = (5, 1, 9)$ and orthogonal to both $\boldsymbol{d}_1 = (1, 0, 1)$ and $\boldsymbol{d}_2 = (2, 0, 3)$.

g. Passes through the point of intersection and orthogonal of both lines, where $\boldsymbol{x} = (2, 1, 1) + t(5, 1, -2)$ and $\boldsymbol{x} = (-2, -1, 2) + t(3, 1, -1)$.

h. Passes through the point of intersection and orthogonal to both lines, where

$$\boldsymbol{x} = \begin{cases} x = t \\ y = -2 + 2t \\ z = 1 + t \end{cases} \quad \text{and} \quad \boldsymbol{x} = \begin{cases} x = 2 + t \\ y = 2 - t \\ z = 3 + 2t \end{cases}.$$

i. Passes through $P = (1, 1)$, parallel to $\boldsymbol{d} = (2, 3)$.

j. Passes through $P = (-2, 5)$, parallel to $\boldsymbol{d} = (0, 1)$.

4.2 [**GHC**] Determine if the described lines are the same line, parallel lines, intersecting or skew lines. If intersecting, give the point of intersection.

a. $\boldsymbol{x} = (1, 2, 1) + t(2, -1, 1)$ and $\boldsymbol{x} = (3, 3, 3) + t(-4, 2, -2)$

b. $\boldsymbol{x} = (2, 1, 1) + t(5, 1, 3)$ and $\boldsymbol{x} = (14, 5, 9) + t(1, 1, 1)$

c. $\boldsymbol{x} = (3, 4, 1) + t(2, -3, 4)$ and $\boldsymbol{x} = (-3, 3, -3) + t(3, -2, 4)$

d. $\boldsymbol{x} = (1, 1, 1) + t(3, 1, 3)$ and $\boldsymbol{x} = (7, 3, 7) + t(6, 2, 6)$

e. $$\boldsymbol{x} = \begin{cases} x = 1 + 2t \\ y = 3 - 2t \\ z = t \end{cases} \quad \text{and} \quad \boldsymbol{x} = \begin{cases} x = 3 - t \\ y = 3 + 5t \\ z = 2 + 7t \end{cases}$$

4.3 [**SM**] For the given point and line, find by projection the point on the line that is closest to the given point, and find the distance from the point to the line.

a. point $(0, 0)$, line $\boldsymbol{x} = (1, 4) + t(-2, 2)$, $t \in \mathbb{R}$

b. point $(2, 5)$, line $\boldsymbol{x} = (3, 7) + t(1, -4)$, $t \in \mathbb{R}$

c. point $(1, 0, 1)$, line $\boldsymbol{x} = (2, 2, -1) + t(1, -2, 1)$, $t \in \mathbb{R}$

d. point $(2, 3, 2)$, line $\boldsymbol{x} = (1, 1, -1) + t(1, 4, 1)$, $t \in \mathbb{R}$

4.4 [**BS**] Find the distance between the origin and the line passing through the points $A(-2, 0, -1)$ and $B(-2, 1, 0)$.

4.5 [**BS**] Given the non-intersecting lines:

$$\begin{aligned} \mathcal{L}_1 &: \quad (x, y, z) = (1, 2, -2) + t_1(1, 2, 1) \\ \mathcal{L}_2 &: \quad (x, y, z) = (2, 1, 3) + t_2(1, 2, 3) \text{ where } t_1,\ t_2 \in \mathbb{R}. \end{aligned}$$

Find the shortest distance between $\mathcal{L}_1$ and $\mathcal{L}_2$.

4.6 [**AG**] Find the distance between the lines

$$L_1 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + r \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{ and } L_2 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}.$$

4.7 [**GHC**] Find the distance between the two lines.

a. $\boldsymbol{x} = (1, 2, 1) + t(2, -1, 1)$ and $\boldsymbol{x} = (3, 3, 3) + t(4, 2, -2)$.

b. $\boldsymbol{x} = (0, 0, 1) + t(1, 0, 0)$ and $\boldsymbol{x} = (0, 0, 3) + t(0, 1, 0)$.

4.8 [**BS**] Find the distance between the line $\mathcal{L} : (x, y, z) = (1, 3, 0) + t(4, 3, 1)$ and the y-axis.

4.9 [**BS**] Given the lines:

$$\begin{array}{lclcl} \mathcal{L}_1 & : & (x, y, z) = (1, 2, -2) & + & t_1(1, 2, 1) \\ \mathcal{L}_2 & : & (x, y, z) = (2, 1, 3) & + & t_2(1, 2, 3) \\ \mathcal{L}_3 & : & (x, y, z) = (1, 1, 1) & + & t_3(2, 7, 3) \end{array}$$

where $t_1,\ t_2,\ t_3 \in \mathbb{R}$. Find the equation of the line which is parallel to $\mathcal{L}_3$ and which intersects both $\mathcal{L}_1$ and $\mathcal{L}_2$.

5 Planes

5.1 [**GHC**] Give the equation of the described plane in standard and general forms.

a. Passes through $(2, 3, 4)$ and has normal vector $\boldsymbol{n} = (3, -1, 7)$.

b. Passes through $(1, 3, 5)$ and has normal vector $\boldsymbol{n} = (0, 2, 4)$.

c. Passes through the points $(1, 2, 3)$, $(3, -1, 4)$ and $(1, 0, 1)$.

d. Passes through the points $(5, 3, 8)$, $(6, 4, 9)$ and $(3, 3, 3)$.

e. Contains the intersecting lines $\boldsymbol{x} = (2, 1, 2) + t(1, 2, 3)$ and $\boldsymbol{x} = (2, 1, 2) + t(2, 5, 4)$.

f. Contains the intersecting lines $\boldsymbol{x} = (5, 0, 3) + t(-1, 1, 1)$ and $\boldsymbol{x} = (1, 4, 7) + t(3, 0, -3)$.

g. Contains the parallel lines $\boldsymbol{x} = (1, 1, 1) + t(1, 2, 3)$ and $\boldsymbol{x} = (1, 1, 2) + t(1, 2, 3)$.

h. Contains the parallel lines $\boldsymbol{x} = (1, 1, 1) + t(4, 1, 3)$ and $\boldsymbol{x} = (2, 2, 2) + t(4, 1, 3)$.

i. Contains the point $(2, -6, 1)$ and the line
$$\boldsymbol{x} = \begin{cases} x = 2 + 5t \\ y = 2 + 2t \\ z = -1 + 2t \end{cases}$$

j. Contains the point $(5, 7, 3)$ and the line
$$\boldsymbol{x} = \begin{cases} x = t \\ y = t \\ z = t \end{cases}$$

k. Contains the point $(5, 7, 3)$ and is orthogonal to the line $\boldsymbol{x} = (4, 5, 6) + t(1, 1, 1)$.

l. Contains the point $(4, 1, 1)$ and is orthogonal to the line
$$\boldsymbol{x} = \begin{cases} x = 4 + 4t \\ y = 1 + 1t \\ z = 1 + 1t \end{cases}$$

m. Contains the point $(-4, 7, 2)$ and is parallel to the plane $3(x - 2) + 8(y + 1) - 10z = 0$.

n. Contains the point $(1, 2, 3)$ and is parallel to the plane $x = 5$.

5.2 [**SM**] Determine the scalar equation of the plane with the given vector equation.

a. $\boldsymbol{x} = (1, 4, 7) + s(2, 3, -1) + t(4, 1, 0),\ s, t \in \mathbb{R}$

b. $\boldsymbol{x} = (2, 3, -1) + s(1, 1, 0) + t(-2, 1, 2),\ s, t \in \mathbb{R}$

c. $\boldsymbol{x} = (1, -1, 3) + s(2, -2, 1) + t(0, 3, 1),\ s, t \in \mathbb{R}$

5.3 [**GHC**] Find the point of intersection between the line and the plane.

a. line: $(1, 2, 3) + t(3, 5, -1)$, plane: $3x - 2y - z = 4$

b. line: $(1, 2, 3) + t(3, 5, -1)$, plane: $3x - 2y - z = -4$

c. line: $(5, 1, -1) + t(2, 2, 1)$, plane: $5x - y - z = -3$

d. line: $(4, 1, 0) + t(1, 0, -1)$, plane: $3x + y - 2z = 8$

5.4 [**SM**] Given the plane $2x_1 - x_2 + 3x_3 = 5$, for each of the following lines, determine if the line is parallel to the plane, orthogonal to the plane, or neither parallel nor orthogonal. If the answer is "neither", determine the angle between the direction vector of the line and the normal vector of the plane.

a. $\boldsymbol{x} = (3, 0, 4) + t(-1, 1, 1),\ t \in \mathbb{R}$

b. $\boldsymbol{x} = (1, 1, 2) + t(-2, 1, -3),\ t \in \mathbb{R}$

c. $\boldsymbol{x} = (3, 0, 0) + t(1, 1, 2),\ t \in \mathbb{R}$

d. $\boldsymbol{x} = (-1, -1, 2) + t(4, -2, 6),\ t \in \mathbb{R}$

e. $\boldsymbol{x} = t(0, 3, 1),\ t \in \mathbb{R}$

5.5 [**AG**] Find the closest point to the point $P = (1, 1, -2)$ on the plane Π of equation $2x - 3y + z = 1$.

5.6 [**YL**] Find the closest point on the plane $2x + y - 3z = 1$ to the point $P(3, 26, 1)$.

5.7 [**SM, YL**] Use a projection (onto or perpendicular to) to find the distance from the point to the plane. And find the closest point on the plane to the given point.

a. point $(-1, -1, 1)$, plane $2x_1 - x_2 - x_3 = 4$

b. point $(0, 2, -1)$, plane $2x_1 - x_3 = 5$

c. point $(2, 3, 1)$, plane $3x_1 - x_2 + 4x_3 = 5$

d. point $(-2, 3, -1)$, plane $2x_1 - 3x_2 - 5x_3 = 5$

5.8 [**BS**] Find the point P on the plane $(x,\ y,\ z) = (0,\ 1,\ 4) + s(0,\ 6,\ -4) + t(-1,\ -1,\ 1)$ where $s, t \in \mathbb{R}$ which is closest to the the origin.

5.9 [**SM**] Determine a vector equation of the line of intersection of the given planes.

a. $x + 3y - z = 5$ and $2x - 5y + z = 7$

b. $2x - 3z = 7$ and $y + 2z = 4$

5.10 [**BS**] Find all unit vectors parallel to the plane $x+2y+3z = 5$ and the xy-plane.

5.11 [**BS**] Determine whether the following planes $x + y + 2z = 5$ and $(x, y, z) = (1, 2, 3) + s(1, 1, 2) + t(3, 0, 7)$ are perpendicular.

5.12 [**BS**] Given the lines $\mathcal{L}_1 :\ (x, y, z) = (1, 3, 0)+t(4, 3, 1)$, $\mathcal{L}_2 :\ (x, y, z) = (1, 2, 3) + t(8, 6, 2)$, and the plance $\mathcal{P} :\ 2x - y + 3z = 15$ and the point $A(1, 0, 7)$.

a. Show that the lines $\mathcal{L}_1$ and $\mathcal{L}_2$ lie in the same plane and find the general equation of this plane.

b. Find the point B on the plane $\mathcal{P}$ which is closest to the point A.

5.13 [**AG**] Let Π be the plane in $\mathbb{R}^3$ with normal vector

$$\boldsymbol{n} = \begin{bmatrix} \cos\theta \\ \cos\theta + \sin\theta \\ \sin\theta \end{bmatrix}.$$

Find all the values of θ in the interval $[0, 2\pi)$ such that Π is parallel to the plane of vector equation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + r\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + t\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$$

.

5.14 [**MB**] Given the following: parametric equations of two skew lines L_1 and L_2, two nonparallel planes P_1 and P_2, coordinates of the point Q, that does not lie on L_1, L_2, P_1, nor P_2.

a. Does a line that is perpendicular to L_1 and passes through Q exist? If so, is it unique? How would the equation of such a line (if it exist), be obtained with the provided information?

b. Does a line parallel to the intersection of the planes P_1 and P_2 and passes through Q exist? If so, is it unique? How would the equation such a line (if it exist), be obtained with the provided information?

c. Does a plane containing L_1 and L_2 exists? Is so, is it unique? How would the equation of such a plane (if it exists), be obtained with the provided information?

5.15 [**BS**] Given the points $A(-3, 0, -1)$ and $B(-2, 1, 0)$.

a. Find the point C on the yz-plane such that the points A, B and C are collinear.

b. Determine whether the points A and B are on the same side of the plane $7x + y + z = 1$.

5.16 [**BS**] Determine whether the point $(1,\ 2,\ 1)$ is between planes $6x - 3y + 6z = -3$ and $4x - 2y + 4z = 2$.

5.17 [**BS**] Given the line $\mathcal{L} :\ (x,\ y,\ z) = (2,\ 2,\ 3) + t(1,\ -1,\ -3)$ where $t \in \mathbb{R}$, the plane $\mathcal{P} : 3x - 2y + 2z = 7$ and the point $A(1,\ 1,\ 1)$.

a. Find parametric equations of the line which contains A, intersects $\mathcal{L}$ and which is parallel to $\mathcal{P}$.

b. Find parametric equations of the line which contains A and which intersects $\mathcal{L}$ at a right angle.

6 Answers to Exercises

Note that either a hint, a final answer or a complete solution is provided.

1.1 $X = \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}$

1.2

a. $X = \begin{bmatrix} 8 & 2 \\ 11 & 4 \end{bmatrix}$ **b.** $X = \begin{bmatrix} 9 & 3 \\ 12 & 3 \end{bmatrix}$

1.3 $A = \begin{bmatrix} 0 & -1 \\ -11 & -\frac{17}{2} \end{bmatrix}$

1.4 $X = \begin{bmatrix} -1 & -2 \\ -6 & -3 \end{bmatrix}$

1.5 $A = \begin{bmatrix} -\frac{3}{4} & 3 \\ 1 & -\frac{3}{4} \end{bmatrix}$

1.6 $A = \begin{bmatrix} 3 & \frac{7}{3} \\ -11 & -\frac{28}{3} \end{bmatrix}$

1.7 $X = \begin{bmatrix} 0 & 0 & 0 \\ -6 & -3 & 0 \\ 0 & -3 & -4 \end{bmatrix}$

1.8 $A = \begin{bmatrix} 1 & 0 \\ 7 & 3 \end{bmatrix}$

1.9 $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{7}{3} & 0 & 1 \end{bmatrix}$

2.1

a. $\operatorname{adj}(A) = \begin{bmatrix} 3 & -4 & 5 \\ -6 & 3 & -5 \\ 5 & -5 & 5 \end{bmatrix}$

b. $\det(A) = -5$

c. $M_{12} = 6$ and $M_{22} = 3$

d. $A^{-1} = \frac{1}{-5}\begin{bmatrix} 3 & -4 & 5 \\ -6 & 3 & -5 \\ 5 & -5 & 5 \end{bmatrix}$

2.2

a. 12

b. $\frac{3^9}{4^3}$

2.3 375

2.4 $\det(M) = \frac{4}{3}$

2.5 $5^3 2^{12}$

2.6

a. -60

b. $-\frac{2^7 5^{12}}{60^3}$

c. $-3^3 5^{16} 60$

2.7 4

2.8 4

2.9 Hint: Use $\operatorname{adj}(A) = (\det A)\, A^{-1}$.

3.1

a. 3

b. $\sqrt{19}$

3.2 $p = -1$

3.3 Expand the left side of the equation by using the fact that $\|\boldsymbol{v}\|^2 = \boldsymbol{v} \bullet \boldsymbol{v}$ for any vector $\boldsymbol{v}$ to get to the right side.

3.4 Where $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^n$, the vectors $\boldsymbol{u} + \boldsymbol{v}$ and $\boldsymbol{u} - \boldsymbol{v}$ are perpendicular if and only if $0 = (\boldsymbol{u} + \boldsymbol{v}) \bullet (\boldsymbol{u} - \boldsymbol{v}) = \boldsymbol{u} \bullet \boldsymbol{u} - \boldsymbol{v} \bullet \boldsymbol{v}$, which shows that those two are perpendicular if and only if $\boldsymbol{u} \bullet \boldsymbol{u} = \boldsymbol{v} \bullet \boldsymbol{v}$. That holds if and only if $\|\boldsymbol{u}\| = \|\boldsymbol{v}\|$.

4.1

a. vector: $\boldsymbol{x} = (2, -4, 1) + t(9, 2, 5)$
parametric: $x = 2 + 9t$, $y = -4 + 2t$, $z = 1 + 5t$
symmetric: $(x - 2)/9 = (y + 4)/2 = (z - 1)/5$

b. vector: $\boldsymbol{x} = (6, 1, 7) + t(-3, 2, 5)$
parametric: $x = 6 - 3t$, $y = 1 + 2t$, $z = 7 + 5t$
symmetric: $-(x - 6)/3 = (y - 1)/2 = (z - 7)/5$

c. Answers can vary: vector: $\boldsymbol{x} = (2, 1, 5) + t(5, -3, -1)$
parametric: $x = 2 + 5t$, $y = 1 - 3t$, $z = 5 - t$
symmetric: $(x - 2)/5 = -(y - 1)/3 = -(z - 5)$

d. Answers can vary: vector: $\boldsymbol{x} = (1, -2, 3) + t(4, 7, 2)$
parametric: $x = 1 + 4t$, $y = -2 + 7t$, $z = 3 + 2t$
symmetric: $(x - 1)/4 = (y + 2)/7 = (z - 3)/2$

e. Answers can vary; here the direction is given by $\boldsymbol{d}_1 \times \boldsymbol{d}_2$:
vector: $\boldsymbol{x} = (0, 1, 2) + t(-10, 43, 9)$
parametric: $x = -10t$, $y = 1 + 43t$, $z = 2 + 9t$
symmetric: $-x/10 = (y - 1)/43 = (z - 2)/9$

f. Answers can vary; here the direction is given by $\boldsymbol{d}_1 \times \boldsymbol{d}_2$:
vector: $\boldsymbol{x} = (5, 1, 9) + t(0, -1, 0)$
parametric: $x = 5$, $y = 1 - t$, $z = 9$
symmetric: not defined, as some components of the direction are 0.

g. Answers can vary; here the direction is given by $\boldsymbol{d}_1 \times \boldsymbol{d}_2$:
vector: $\boldsymbol{x} = (7, 2, -1) + t(1, -1, 2)$
parametric: $x = 7 + t$, $y = 2 - t$, $z = -1 + 2t$
symmetric: $x - 7 = 2 - y = (z + 1)/2$

h. Answers can vary; here the direction is given by $\boldsymbol{d}_1 \times \boldsymbol{d}_2$:
vector: $\boldsymbol{x} = (2, 2, 3) + t(5, -1, -3)$
parametric: $x = 2 + 5t$, $y = 2 - t$, $z = 3 - 3t$
symmetric: $(x - 2)/5 = -(y - 2) = -(z - 3)/3$

i. vector: $\boldsymbol{x} = (1, 1) + t(2, 3)$
parametric: $x = 1 + 2t$, $y = 1 + 3t$
symmetric: $(x - 1)/2 = (y - 1)/3$

j. vector: $\boldsymbol{x} = (-2, 5) + t(0, 1)$
parametric: $x = -2$, $y = 5 + t$
symmetric: not defined

4.2

a. Parallel.

b. Intersecting; $(12, 3, 7)$.

c. Intersecting; $(9, -5, 13)$.

d. Parallel.

4.3

a. $\left(\frac{5}{2}, \frac{5}{2}\right)$, $\frac{5}{\sqrt{2}}$

b. $\left(\frac{58}{17}, \frac{91}{17}\right)$, $\frac{6}{\sqrt{17}}$

c. $\left(\frac{17}{6}, \frac{1}{3}, -\frac{1}{6}\right)$, $\sqrt{\frac{29}{6}}$

d. $\left(\frac{5}{3}, \frac{11}{3}, -\frac{1}{3}\right)$, $\sqrt{6}$

4.4 $\frac{3}{\sqrt{2}}$

4.5 $\frac{3}{5}\sqrt{5}$

4.6 $\sqrt{6}/6$

4.7

a. $3/\sqrt{2}$

b. 2

4.8 $\frac{1}{\sqrt{17}}$

4.9 $(x,\ y,\ z) = (\frac{3}{2},\ 3,\ -\frac{3}{2}) + t(2,\ 7,\ 3)$ where $t \in \mathbb{R}$

5.1

a. Standard form: $3(x - 2) - (y - 3) + 7(z - 4) = 0$
general form: $3x - y + 7z = 31$

b. Standard form: $2(y - 3) + 4(z - 5) = 0$
general form: $2y + 4z = 26$

c. Answers may vary;
Standard form: $8(x - 1) + 4(y - 2) - 4(z - 3) = 0$
general form: $8x + 4y - 4z = 4$

d. Answers may vary;
Standard form: $-5(x-5)+3(y-3)+2(z-8)=0$
general form: $-5x+3y+2z=0$

e. Answers may vary;
Standard form: $-7(x-2)+2(y-1)+(z-2)=0$
general form: $-7x+2y+z=-10$

f. Answers may vary;
Standard form: $3(x-5)+3(z-3)=0$
general form: $3x+3z=24$

g. Answers may vary;
Standard form: $2(x-1)-(y-1)=0$
general form: $2x-y=1$

h. Answers may vary;
Standard form: $2(x-1)+(y-1)-3(z-1)=0$
general form: $2x+y-3z=0$

i. Answers may vary;
Standard form: $2(x-2)-(y+6)-4(z-1)=0$
general form: $2x-y-4z=6$

j. Answers may vary;
Standard form: $4(x-5)-2(y-7)-2(z-3)=0$
general form: $4x-2y-2z=0$

k. Answers may vary;
Standard form: $(x-5)+(y-7)+(z-3)=0$
general form: $x+y+z=15$

l. Answers may vary;
Standard form: $4(x-4)+(y-1)+(z-1)=0$
general form: $4x+y+z=18$

m. Answers may vary;
Standard form: $3(x+4)+8(y-7)-10(z-2)=0$
general form: $3x+8y-10z=24$

n. Standard form: $x-1=0$
general form: $x=1$

5.2

a. $x-4y-10z=-85$

b. $2x-2y+3z=-5$

c. $-5x-2y+6z=15$

5.3

a. No point of intersection; the plane and line are parallel.

b. The plane contains the line, so every point on the line is a "point of intersection."

c. $(-3,-7,-5)$

d. $(3,1,1)$

5.4

a. The line is parallel to the plane.

b. The line is orthogonal to the plane.

c. The line is neither parallel nor orthogonal to the plane, $\theta \approx 0.702$ radians.

d. The line is orthogonal to the plane.

e. The line is parallel to the plane.

5.5 $(11/7,\ 1/7,\ -12/7)$

5.6 $(-1,\ 24,\ 7)$

5.7

a. $\sqrt{6}$, $(1,-2,0)$

b. $\frac{4}{\sqrt{5}}$, $\left(\frac{8}{5},2,-\frac{9}{5}\right)$

c. $\frac{2}{\sqrt{26}}$, $\left(\frac{23}{13},\frac{40}{13},\frac{9}{13}\right)$

d. $\frac{13}{\sqrt{38}}$, $\left(-\frac{25}{19},\frac{75}{38},-\frac{103}{38}\right)$

5.8 $(1,\ 2,\ 3)$

5.9

a. $\boldsymbol{x}=\left(\frac{46}{11},\frac{3}{11},0\right)+t(-2,-3,-11)$, $t\in\mathbb{R}$

b. $\boldsymbol{x}=\left(\frac{7}{2},4,0\right)+t(3,-4,2)$, $t\in\mathbb{R}$

5.10 $\pm\left(\frac{2}{\sqrt{5}},\ \frac{-1}{\sqrt{5}},\ 0\right)$.

5.11 The two planes are perpendicular.

5.12

a. Since the two lines are parallel they lie on the same plane. $5x-6y-2x-13=0$.

b. $B(-\frac{1}{7},\frac{4}{7},\frac{37}{7})$

5.13 $0,\ \pi$

5.14

a. Such a line exists, but it is not unique. One of such line is obtained by finding a vector $\boldsymbol{d}$ perpendicular to the direction vector of L_1, then the equation of the line is given by $\boldsymbol{x}=Q+t\boldsymbol{d}$ where $t\in\mathbb{R}$.

b. Such a line exists and is unique. The line can be obtained by finding a vector $\boldsymbol{d}$ parallel to the intersection of L_1 and L_2, then the equation of the line is given by $\boldsymbol{x}=Q+t\boldsymbol{d}$ where $t\in\mathbb{R}$.

c. Such a plane does not exist since the lines are skew.

5.15

a. $(0,\ 3,\ 2)$

b. Yes, the points are on the same side.

5.16 The point is not in between the planes.

5.17

a. $x=1+6t,\ y=1-4t,\ z=1-13t$ where $t\in\mathbb{R}$

b. $x=1+17t,\ y=1+5t,\ z=1+4t$ where $t\in\mathbb{R}$

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