Dawson College: Winter 2020: Linear Algebra (SCIENCE): 201-NYC-05-S3: Test 1, part 1 of 2 name:
Books, watches, notes or cell phones are not allowed. The only calculators allowed are the Sharp EL-531. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.
Question 1. ¹ (1 mark each) Complete each of the following sentences with MUST, MIGHT, or CANNOT.
a. Let A be a square matrix. If $A\mathbf{x} = A\mathbf{y}$ for distinct \mathbf{x} and \mathbf{y} , then A
b. If the 3×3 coefficient matrix A has a RREF with two leading ones, then the system $A\mathbf{x} = \mathbf{b}$ be inconsistent when $\mathbf{b} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$.
c. If A, B, and C are square matrices such that $ABC^2 = I$ and A is invertible then matrix B be invertible.
d. If A is a square matrix and $A^4 - 2A^2 + A = I$, then A be invertible.
e. If A and B are $n \times n$ matrices such that $AB = B$, then A be an identity matrix.
f. Given an $n \times n$ matrix A . If the system $A\mathbf{x} = \mathbf{b}$ is inconsistent for some $\mathbf{b} \in \mathbb{R}^n$, then the system $A\mathbf{x} = 0$ have non-trivial solutions.
g. If E_1 and E_2 are two elementary matrices, then E_1E_2 be equal to E_2E_1 .
h. The expression $(I-A)(I+A)$ be equal to $I-A^2$.
i. For any invertible matrix A , the number of leading ones of the RREF of A be the same as the number of leading ones of the RREF of A^2 .
j. If E_1 , E_2 are elementary matrices, then E_1E_2 also be an elementary matrix.
k. If matrix AB is invertible, then A be invertible
Question 2. ¹ (2 marks) Suppose u is a solution to A x = b and v is a solution to A x = 0 . Show that w = 3 u - 4 v is a solution to A x = 3 b .
Question 3. (1.5 marks) Illustrate all relative positions of lines in an inconsistent linear system of three lines.

Question 4. (4 marks) Find all 2×2 symmetric matrices A such that $A\begin{bmatrix} 1 & -2 \end{bmatrix}^T - \begin{bmatrix} 3 & 4 \end{bmatrix}^T = \mathbf{0}$

 $^{^{\}rm 1}$ From or modified from a John Abbott final examination

Question 5.1 (5 marks) Let
$$A = \begin{bmatrix} 1 & 1 & a \\ 1 & a & a \\ a & a & a^2 \end{bmatrix}$$
 and $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ a^2 - 2a \end{bmatrix}$. Find the value(s) of a , if possible, for which the equation $A\mathbf{x} = \mathbf{b}$

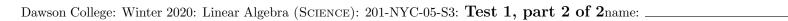
has:

a. a unique solution,

b. infinitely many solutions,

c. no solution.

Question 6. (5 marks) Consider the matrix equation
$$E_1^{-1}E_2 = (E_3 - \text{tr}(I_5)A)^{-1}$$
 where $E_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $E_2 = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$. Solve for A .



Question 7. (5 marks) Find elementary matrices
$$E_1, E_2$$
 and E_3 which satisfy the following equation: $E_3E_2E_1\begin{bmatrix}0 & 1 & 0\\ 5 & 0 & 0\\ 0 & 1 & 1\end{bmatrix} = I$

Question 8. (3 marks each) A square matrix A is idempotent if $A^2 = A$.

- a. Prove that if A and B are idempotent and are commutative, then AB is idempotent.
- b. Prove that if A is idempotent then either A is singular or A = I. Hint: Prove by contradiction.

Question 9. (5 marks) Given A an $n \times n$ invertible matrix. Show that the reduced row echelon form B is I_n if and only if $(AB)\mathbf{x} = \mathbf{0}$ has only the trivial solution.

Bonus Question. (5 marks) Enumerate all the solution(s) of $A\mathbf{x} = \mathbf{0}$ where

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 2 & 0 & 2 \end{bmatrix}$$

given that the numbers are \mathbb{Z}_3 instead of \mathbb{R} . Operations on the numbers of \mathbb{Z}_3 can be defined by the following Cayley tables:

	0					1	
0	0	1	2			0	
1	1	2	0	1	0	1	2
2	2	0	1	2	0	2	1