Dawson College: Winter 2020: Linear Algebra (SCIENCE): 201-NYC-05-S3: **Test 2** name: ____________________

No books, watches, notes or cell phones are allowed. The **only** calculators allowed are the Sharp EL-531. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(1 mark per blank)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. If A is a product of elementary matrices, then $\det(A)$ ____________ equal zero.

b. Let A be a 3×3 matrix, and let B be a 4×4 matrix. If the leading ones of the RREF of A is equal to those of the RREF of B, then $\det(A)$ ____________ equal zero and det(B) ____________ equal zero.

Question 2.[1] *(1 mark per blank)* Let A and B be invertible $n \times n$ matrices. Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. $A + B$ ____________ be invertible.

b. $A^T A$ ____________ be invertible.

Question 3. *(1 mark per blank)* Given A an $n \times n$ matrix and k a non-zero scalar.

a. If A is an elementary matrix obtained by interchanging two rows then $\det(A) =$ ____________.

b. If A is the reduced row echelon form of an invertible matrix then $\det(A) =$ ____________.

c. If A is a singular matrix then $\det(A) =$ ____________.

d. If A is an elementary matrix obtained by adding k times one row to an other then $\det(A) =$ ____________.

e. If A is an elementary matrix obtained by multiplying one row by k then $\det(A) =$ ____________.

f. If A is the identity matrix multiplied by k then $\det(A) =$ ____________.

Question 4.[1] Suppose A is an $n \times n$ matrix such that $\det(A) = 5$ and $\det(2A^T) = 40$.

a. *(3 marks)* Find n.

b. *(2 marks)* Find $\det(\operatorname{adj}(3A))$.

Question 5.[1] *(5 marks)* Given that $\det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right) = 10$ and $A = \begin{bmatrix} 3g+a & 3h+b & 2 & 3i+c \\ d+2a & e+2b & 3 & f+2c \\ a & b & 4 & c \\ 0 & 0 & 5 & 0 \end{bmatrix}$

a. *(5 marks)* Find $\det(A)$.

b. *(3 marks)* Using Cramer's Rule find x_1 and x_3 for $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = \begin{bmatrix} 2a & 3a & 4a & 5a \end{bmatrix}^T$

Question 6. *(2 marks)* Prove: If the entries in each row of an $n \times n$ matrix A add up to zero, then the determinant of A is zero.

[1] From or modified from a John Abbott final examination

Question 7. Given $A^{-1} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ -1 & 0 & 0 & 1 \\ 4 & -2 & 0 & 0 \\ 0 & 0 & 5 & 1 \end{bmatrix}$.

a. *(5 marks)* Find $\det(A)$.

b. *(2 marks)* Find $\text{adj}(A)$.

Question 8.[1] *(3 marks each)* Determine whether the following statements are true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. If A is a symmetric $n \times n$ matrix where n is even then $\det(A) = 0$.

b. If A is a skew-symmetrix $n \times n$ matrix where n is odd then $\det(A) = 0$.

Bonus Question. *(2 marks)* Find $\det(A)$ where

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 2 & 0 & 2 \end{bmatrix}$$

given that the numbers are $\mathbb{Z}_3$ instead of $\mathbb{R}$. Operations on the numbers of $\mathbb{Z}_3$ can be defined by the following Cayley tables:

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

×	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1