Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show and justify all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. If a_n and b_n are divergent, then $a_n + b_n$ ____________ be divergent.

b. If $a_n > 0$ and $\lim_{n\to\infty}(a_{n+1}/a_n) < 1$ then $\lim_{n\to\infty} a_n$ ____________ be equal to zero.

c. If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ ____________ be convergent.

d. A bounded sequence ____________ converge.

e. If $\sum c_n x^n$ diverges when $x = 2$, then it ____________ diverge when $x = 1$.

Question 2. *(5 marks)* Find the sum of the following series:

$$\sum_{n=2}^{\infty} [\operatorname{arcsec}(n) - \operatorname{arcsec}(n+1)]$$

Question 3.[1] *(3 marks)* Let $\sum_{n=1}^{\infty} a_n$ be a series whose n^{th} partial sum is given by $S_n = \frac{2n+1}{n+2}$. Find a_9.

[1]Modified from a John Abbott final examination

Question 4.[2]

a. *(1 mark)* Show that $0 < \frac{(n!)^2}{(2n)!} < 1$ when $n \geq 1$.

b. *(3 marks)* Use part (a) (whether or not you have shown it) to determine if the following sequence converges or diverges. If it converges, find its limit.

$$a_n = \frac{(n!)^2}{(2n+1)!}, \qquad n \geq 1.$$

[2] From a Dawson College final examination

Question 5. *(5 marks)* Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(x+2)^n}{4^n \sqrt[5]{n+3}}$$

Question 6. *(5 marks)* Find the Taylor series for $f(x) = \sqrt{x}$ centered at 9.

Question 7. *(5 marks)* Determine whether the following series converges or diverges. Justify your answers

$$\sum_{n=1}^{\infty} \frac{1 + \arcsin(\frac{1}{n})}{1 + \arctan(n)}$$

Question 8. *(10 marks)* Determine whether the foloowing series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=2022}^{\infty} (-1)^n \frac{\ln(\ln n) + 1}{n \ln n}$$

Bonus Question a. *(3 marks)* Using the $K(\epsilon)$ definition show that the sequence $(-1)^n$ is divergent

Bonus Question b. *(3 marks)* Using the $K(\epsilon)$ definition show that given the sequence a_n and $L \in \mathbb{R}$ such that there exists a $k \in \mathbb{N}$ such that for all $n \geq k$, $a_n = L$ then a_n converges to L.

Bonus Question c. *(5 marks)* Using the $K(\epsilon)$ definition show that given the convergent sequence a_n and $c \in \mathbb{R}$ then

$$\lim_{n\to\infty} ca_n = c \lim_{n\to\infty} a_n$$