Dawson College: Winter 2022: Linear Algebra (Science): 201-NYC-05-S7: **Test 2, part 1 of 2** name: ________________

No books, watches, notes or cell phones are allowed. The **only** calculators allowed are the Sharp EL-531**. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(1 mark per blank)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. If A is a product of elementary matrices, then $\det(A)$ ____________ equal one.

b. Let A be a 3×3 matrix, and let B be a 4×4 matrix. If the leading ones of the RREF of A is equal to those of the RREF of B, then $\det(A)$ ____________ equal zero and det(B) ____________ equal zero.

c. If $\vec{u}$ and $\vec{v}$ are nonzero vectors and $\text{proj}_{\vec{v}}\vec{u} = \vec{u}$, then $\vec{u}$ ____________ be parallel to $\vec{v}$.

d. Let $\vec{w}$ be orthogonal to both $\vec{u}$ and $\vec{v}$. Then $\vec{w}$ ____________ be orthogonal to $\vec{u} + \vec{v}$.

e. The vector $\vec{u} \times (\vec{v} \times \vec{w})$ ____________ be parallel to the vector $\vec{u}$.

f. The vector $\vec{u} \times (\vec{v} \times \vec{w})$ ____________ be orthogonal to the vector $2\vec{v} \times (-4\vec{w})$.

Question 2. *(1 mark per blank)* Given A an $n \times n$ matrix and k a non-zero scalar.

a. If A is the reduced row echelon form of a singular matrix then $\det(A) =$ ____________.

b. If A is an elementary matrix obtained by adding k times one row to an other then $\det(A) =$ ____________.

c. If A is the identity matrix multiplied by k then $\det(A) =$ ____________.

Question 3. *(5 marks)* [2] Given A, an $n \times n$ matrix such that $\det(A) = 9$ and

$$A^3 A^T = 3A^{-1}\text{adj}(A)$$

find n.

Question 4. *(5 marks)* Using elementary operations show that

$$-sr\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} sb + 2d & rsa + 2rc \\ d & rc \end{vmatrix}$$

[1] From or modified from a John Abbott final examination

[2] From a Dawson College final examination

Question 4. Given $A^{-1} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ -1 & 0 & 0 & 1 \\ 4 & -2 & 0 & 0 \\ 0 & 0 & 5 & 1 \end{bmatrix}$.

a. *(5 marks)* Find $\det(A)$.

b. *(2 marks)* Find $\text{adj}(A)$.

Question 5.[1] *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If A is a symmetric $n \times n$ matrix where n is even then $\det(A) = 0$.

Question 6. *(3 marks)* Solve only for x_3 using Cramer's Rule.

$$\begin{cases} x_1 - 2x_2 + 3x_3 = 4 \\ \qquad 5x_2 - 6x_3 = 7 \\ \qquad\qquad 8x_3 = 9 \end{cases}$$

Question 7. *(2 marks)* Using sketches illustrate that vector addition is commutative.

Question 8.[1] Let $\vec{u}$ and $\vec{v}$ be vectors in $\mathbb{R}^n$. Given: $||\vec{u}|| = 5$, $||\vec{u} + 2\vec{v}|| = \sqrt{2}$, $\vec{v}$ and $\vec{u} + 3\vec{v}$ are both unit vectors, and the angle between $\vec{u} + 2\vec{v}$ and $\vec{u} + 3\vec{v}$ is $\pi/4$.

a. *(3 marks)* Find $\vec{u} \cdot \vec{v}$.

b. *(2 marks)* Find $||\vec{u} + \vec{v}||$.

Question 9. Given two planes: $\mathcal{P}_1 :\ x + z = 1$ and $\mathcal{P}_2 :\ y + z = 1$.

a. *(1 mark)* Give a point of intersection of the two planes, by inspection.

b. *(1 mark)* Give a geometrical argument to explain why the intersection of the two planes is a line.

c. *(2 marks)* Find a direction vector for the line of intersection of the two planes without solving for the solution set.

d. *(2 marks)* Find the solution set of the system of linear equations determined by $\mathcal{P}_1$ and $\mathcal{P}_2$ by only using part a) and part c). Justify.

e. *(1 marks)* Sketch a geometrical interpretation to part d).

Question 10. Given the points $A(3, 0, -1)$ and $B(3, 1, 0)$.

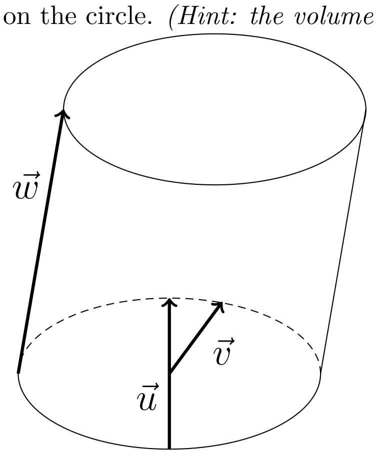
a. *(1 mark)* Find the equation of the line which passes through A and B.

b. *(4 marks)* Find the points on the line which passes through A and B which are $\sqrt{21}$ units away from the origin.

Question 11.[3] *(4 marks)* Determine whether the two lines $\mathcal{L}_1 : \vec{x} = (1 - 2t,\ 2 - t,\ 3 + 3t)$ and $\mathcal{L}_2 : \vec{x} = (4 + 3t,\ -1 + t,\ 2 + t)$ intersect, are parallel or are skew lines.

Question 12. *(5 marks)* A cylinder where the sides are not perpendicular to the base (the circle) is called an *oblique cylinder.* Given the oblique cyclinder defined by the given vectors $\vec{u} = (2, 2, 4)$, $\vec{v} = (1, 2, 1)$ and $\vec{w} = (4, 1, 3)$. Find the volume of the oblique cylinder.

Note that from the diagram that $\vec{w}$ is not perpendicular to the base, that $\vec{v}$ is positioned such that its tail is at the center of the circle and its tip lies on the circle, that $\vec{u}$ is positioned such that the vector passes through the center of the circle while its tail and tip lie on the circle. *(Hint: the volume of an oblique cyclinder is equal to the area of the base times the height.)*



Bonus Question. *(5 marks)* If $\vec{u}, \vec{v} \in \mathbb{R}^n$ then $|\vec{u} \cdot \vec{v}| \leq ||\vec{u}||||\vec{v}||$. *Hint: Analyse the squared norm of* $||\vec{u}||\vec{v} - ||\vec{v}||\vec{u}$ *and* $||\vec{u}||\vec{v} + ||\vec{v}||\vec{u}$.

[3] From the assigned homework.