Dawson College: Winter 2022: Linear Algebra (SCIENCE): 201-NYC-05-S7: **Test 3, part 1 of 2** name: ______________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. If $\vec{v}$ is in Span($\{\vec{u}\}$), then $\vec{u}$ ______________ be in Span($\{\vec{v}\}$).

b. If $A\mathbf{x} = \mathbf{b}$ has two distinct solutions then the columns of A ______________ be linearly dependent

c. The columns of an elementary matrix ______________ form a linearly independent set.

d. If the set $\{\vec{u}, \vec{v}, \vec{w}\}$ spans a plane in $\mathbb{R}^3$, then the set $\{\vec{u}, \vec{v}\}$ ______________ span the same plane.

e. If $\{\vec{u}, \vec{v}\}$ is a basis for a subspace S, then the set $\{6\vec{u} + 3\vec{v}, 10\vec{u} + 5\vec{v}\}$ ______________ also be a basis for S.

Question 2.[1] *(1 mark each)*

a. Suppose that $(3, -2, 7)$ and $(-2, a, b)$ is linearly independent then a possibility for (a, b) is $(a, b) =$ ______________.

b. The vector space of all skew-symmetric $n \times n$ matrices has dimension ______________.

Question 3.[1] Consider the set $H = \left\{ A \mid A \in \mathcal{M}_{2\times 2} \text{ and } \begin{bmatrix} 1 & 3 \end{bmatrix} A \begin{bmatrix} 1 & 3 \end{bmatrix}^T = \mathbf{0} \right\}$.

a. *(1 marks)* Find two vectors of H.

b. *(4 marks)* Show that H is a subspace of $\mathcal{M}_{2\times 2}$.

[1] From or modified from a John Abbott final examination

Question 4.[2] Let $V = \{(a,b) \mid a,b \in \mathbb{R}, b > 0\}$. And the addition in V is defined by $(a,b) \bigoplus (c,d) = (ad+bc, bd)$ and scalar multiplication in V is defined by $t \bigodot (a,b) = (tab^{t-1}, b^t)$

a. *(1 mark)* $(4,2) \bigoplus (-5,1)$

b. *(1 mark)* $-2 \bigodot (1,2)$

c. *(3 marks)* Demonstrate whether the 4th axiom of vector spaces holds. That is, does the zero vector exists.

Question 5.[3] *(7 marks)* Find a basis for all vectors of the form (a,b,c,d) where $c = a+b$ and $d = a-b$ and state its dimension. Also express $(2,3,-1,5)$ relative to the basis found, if possible.

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[2]From http://www.math.uwaterloo.ca/ jmckinno/Math225/Week1/Lecture1e.pdf

[3]From the assigned homework.

Question 6.[3] *(4 marks)* Prove: Let $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ be a basis for a vector space V. Show that $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ is also a basis, where $\vec{u}_1 = \vec{v}_1$, $\vec{u}_2 = \vec{v}_1 + \vec{v}_2$, and $\vec{u}_3 = \vec{v}_1 + \vec{v}_2 + \vec{v}_3$.

Question 7. *(4 marks)* Determine whether $\left\{ \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ 7 & 3 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ -3 & 5 \end{bmatrix} \right\}$ is linearly independent.

Question 8. *(4 marks)* In any vector space V, for any $\vec{u}, \vec{v}, \vec{w} \in V$ prove that if $\vec{v} + \vec{w} = \vec{u} + \vec{w}$ then $\vec{v} = \vec{u}$. *Show every step, justify every step, and cite the axiom(s) used!!!*

Bonus Question. *(1+5 marks)* State and prove the Exchange Lemma.