Dawson College: Winter 2022: Linear Algebra (SCIENCE): 201-NYC-05-S8: **Test 1, part 1 of 2** name: ____________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. A system of linear equations with more equations than variables ______________ have a unique solution.

b. If the 3×3 coefficient matrix A has a RREF with a single leading one, then the system $A\mathbf{x} = \mathbf{b}$ ______________ be inconsistent when $\mathbf{b} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$.

c. If the bottom row of a matrix in reduced row-echelon form contains all 0's to the left of the vertical bar and a nonzero entry to the right, then the system ______________ have no solution.

d. If A is a square matrix and $A^4 - 2A^2 + A = 0$, then A ______________ be invertible.

e. Given an $n \times n$ matrix A. If the system $A\mathbf{x} = \mathbf{b}$ is consistent for all $\mathbf{b} \in \mathbb{R}^n$, then the system $A\mathbf{x} = \mathbf{0}$ ______________ have non-trivial solutions.

f. The expression $(B - A)(B + A)$ ______________ be equal to $B^2 - A^2$ if A and B are commutative.

g. If E_1, E_2 are elementary matrices, then E_1E_2 ______________ also be an elementary matrix.

h. If matrix AB is invertible, then B ______________ be invertible.

Question 2.[1] *(5 marks)* Given X, A, B, and C are $n \times n$ matrices, solve the following equation for X. Assume any necessary matricesto be invertible.

$$2X^T + A = (XB - C)^T$$

Question 3.[2] *(5 marks)*Find **all** matrices A, if any, such that $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} A = I_2$

[1]From or modified from a John Abbott final examination or WeBWorK

[2]From a Dawson final examination

Question 4. Given

$$A = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 2 & -2 & 0 & 3 \\ 3 & 0 & 0 & -2 \\ 5 & -2 & -1 & 0 \end{bmatrix}$$

a. *(4 marks)* Find two row echelon form of the matrix A.

b. *(2 marks)* Suppose that A is the augmented matrix of a linear system. Write the corresponding system of linear equations. Use one of the REF from part a. to find the solution of the system.

c. *(1 mark)* Express A as a product of elementary matrices, if possible. Justify.

d. *(1 mark)* Dethermine whether A is symmetric, skew-symmetric or neither. Justify.

Question 5a.[1] *(5 marks)*Let $A = \begin{bmatrix} 1 & 0 & 5 \\ -1 & 3 & 1 \\ 1 & k & 12 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -2 \\ 8 \\ h \end{bmatrix}$. Find the value(s) of h and k, if possible, for which the equation $A\mathbf{x} = \mathbf{b}$ has:

a. a unique solution,

b. infinitely many solutions,

c. no solution.

Question 5b. *(3 marks)* Find the solution set in the case where the above system has infinitely many solutions. Give a particular solution. And the particular solution when the second variable is equal to zero.

Question 6. *(3 marks)* A square matrix A is *idempotent* if $A^2 = A$. Prove that if A is idempotent then $(I - 2A) = (I - 2A)^{-1}$.

Question 7. *(5 marks)* Express $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and its inverse as a product of elementary matrices, if possible. Justify.

Question 8.[3] *(5 marks)* Let $A\mathbf{x} = \mathbf{0}$ be a homogeneous system of n linear equations in n unknowns, and let Q be an invertible $n \times n$ matrix. Prove that $A\mathbf{x} = \mathbf{0}$ has only the trivial solution if and only if $(QA)\mathbf{x} = \mathbf{0}$ has only the trivial solution.

Bonus Question. *(5 marks)* Prove: $(aA)(bB) = abAB$ where A and B are matrices such that their multiplication is defined and $a, b \in \mathbb{R}$

[3] From assigned homework.