Dawson College: Winter 2022: Linear Algebra (SCIENCE): 201-NYC-05-S8: **Test 3, part 1 of 2** name: ______________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. Let $S = \{\vec{u}, \vec{v}\}$ be a set of vectors. If $\vec{w}$ is in Span(S), then $\vec{w}$ ______________ be in S.

b. Let $\vec{u}, \vec{v}$, and $\vec{w}$ be distinct nonzero vectors in $\mathbb{R}^3$. If $\{\vec{u}, \vec{v}, \vec{w}\}$ is linearly independent, then $\vec{u} \cdot (\vec{v} \times \vec{w})$ ______________ be equal to $\vec{u} \cdot (\vec{w} \times \vec{v})$.

c. If $\{\vec{a}, \vec{b}, \vec{c}\}$ is a linearly independent set in Span$(\{\vec{u}, \vec{v}, \vec{w}\})$, then $\{\vec{u}, \vec{v}, \vec{w}\}$ ______________ be a linearly independent set.

d. If B has no column of zeros, but AB does, then the columns of A ______________ be linearly independent.

e. If the column vectors of a square matrix A span all of $\mathbb{R}^3$, then the determinant of A ______________ be zero.

Question 2.[1] *(1 mark each)*

a. Suppose that $(3, -2, 7)$ and $(-2, a, b)$ is linearly dependent then $(a, b) =$ ______________.

b. The vector space of all symmetric $n \times n$ matrices has dimension ______________.

Question 3.[1] Consider the subspace $H = \left\{ A \mid A \in \mathcal{M}_{2\times 2} \text{ and } \begin{bmatrix} 1 & 3 \end{bmatrix} A \begin{bmatrix} 1 & 3 \end{bmatrix}^T = \mathbf{0} \right\}$.

a. *(1 marks)* Find two vectors of H.

b. *(4 marks)* Find a basis for H.

c. *(1 mark)* State the dim(H).

d. *(2 marks)* Express $\begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}$ relative to the basis found in part b., if possible.

[1] From or modified from a John Abbott final examination

Question 4.[2] Let $V = \{(a,b) \mid a,b \in \mathbb{R}, b > 0\}$. And the addition in V is defined by $(a,b) \bigoplus (c,d) = (ad+bc, bd)$ and scalar multiplication in V is defined by $t \bigodot (a,b) = (tab^{t-1}, b^t)$

a. *(1 mark)* $(2,3) \bigoplus (-2,1)$

b. *(1 mark)* $-3 \bigodot (3,1)$

c. *(3 marks)* Demonstrate whether the 5th axiom of vector spaces holds given that the $\vec{0}$ is $(0,1)$. That is, do additive inverse exists for all vectors in V.

Question 5.[3] *(4 marks)* Determine whether the set of all $n \times n$ matrices A such that trace$(A) = 0$ is a subspace of $\mathcal{M}_{n\times n}$.

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[2] From http://www.math.uwaterloo.ca/ jmckinno/Math225/Week1/Lecture1e.pdf

[3] From the assigned homework.

Question 6.[3] *(4 marks)* Prove: If $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent and $\vec{v}_3$ does not lie in $\text{span}(\{\vec{v}_1, \vec{v}_2\})$, then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent.

Question 7. *(4 marks)* Determine whether $\left\{p_1(x) = 1 + 2x - 3x^2 + 5x^3, p_2(x) = 2 - x + 7x + 3x^2, p_3(x) = 3 - x + 3x^2 + 5x^3\right\}$ is linearly independent.

Question 8. *(4 marks)* In any vector space V, for any $\vec{u}, \vec{v}, \vec{w} \in V$ prove that if $\vec{v} + \vec{w} = \vec{u} + \vec{w}$ then $\vec{v} = \vec{u}$. *Show every step, justify every step, and cite the axiom(s) used!!!*

Bonus Question. *(1+5 marks)* State and prove the Exchange Lemma.