Webwork - assignment 13 - Problem 17

(1 point) Let w, x, y, z be vectors and suppose $z = -x + 3y$ ① and $w = -3x - 2y - 3z$ ②.

Mark the statements below that must be true.

- [x] **A.** Span(x, y) = Span(x, w, z)
- [ ] **B.** Span(x, z)= Span(y, w)

} solved below

- [ ] **C.** Span(y, w)= Span(z)
- [x] **D.** Span(y) = Span(w)

} Can be solved similarly to part A and B.

apply the following theorem seen in class

Theorem 2.4. *Let S and S' be subsets of a vector space V. If every vector in S is expressible as a linear combination of the vectors in S' then span(S) is a subspace of span(S'). If in addition every vector of S' is expressible as a linear combination of the vectors in S then span(S) = span(S').*

A. Lets determine whether $\text{Span}(x,y) \subseteq \text{Span}(x,w,z)$

$x \in \text{Span}(x,w,z)$ since $x = 1\cdot x + 0\cdot w + 0\cdot z$

$y \in \text{Span}(x,w,z)$ since by ① $z = -x + 3y$

$$3y = x + z$$
$$y = \tfrac{1}{3}x + \tfrac{1}{3}z$$
$$y = \tfrac{1}{3}x + 0\cdot w + \tfrac{1}{3}z$$

$\therefore \text{Span}(x,y) \subseteq \text{Span}(x,w,z)$

Lets determine whether $\text{Span}(x,w,z) \subseteq \text{Span}(x,y)$

$x \in \text{Span}(x,y)$ since $x = 1\cdot x + 0\cdot y$

$w \in \text{Span}(x,y)$ since if we sub ① into ② $w = -3x - 2y - 3(-x+3y)$

$$= -11y$$
$$= 0\cdot x + (-11)\cdot y$$

$z \in \text{Span}(x,y)$ since by ① we have $z = -x + 3y$

$\therefore \text{Span}(x,w,z) \subseteq \text{Span}(x,y)$

$\therefore \text{Span}(x,w,z) = \text{Span}(x,y)$

B. Lets determine whether $\text{Span}(x,z) \subseteq \text{Span}(y,w)$

$x \notin \text{Span}(y,w)$ using ① and ② $\left.\begin{array}{l} z = -x + 3y \\ \text{and } w = -3x - 2y - 3z \end{array}\right\} \Rightarrow \begin{array}{l} x + z - 3y = 0 \\ 3x + 3z + 2y - w = 0 \end{array}$

$$\begin{bmatrix} 1 & 1 & -3 & 0 & 0 \\ 3 & 3 & 2 & -1 & 0 \end{bmatrix}$$

$$\underset{-3R_1 + R_2 \to R_2}{\sim} \begin{bmatrix} 1 & 1 & -3 & 0 & 0 \\ 0 & 0 & 11 & -1 & 0 \end{bmatrix}$$

$$\underset{\frac{3}{11}R_2 + R_1 \to R_1}{\sim} \begin{bmatrix} 1 & 1 & 0 & -\frac{3}{11} & 0 \\ 0 & 0 & 11 & -1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{array}{ll} x + z - \frac{3}{11}w = 0 & ③ \\ 11y - w = 0 & ④ \end{array}$$

which implies that x can be written as a lin. comb. of z and w or as a lin. comb of z and y if we substitute ④ into ③

$\therefore \text{Span}(x,z) \nsubseteq \text{Span}(y,w)$ $\therefore \text{Span}(x,z) \neq \text{Span}(y,w)$,