Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S3: Winter 2023: **Quiz 10** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. For each of the following parts, **set up** an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis using the specified method. Sketch the region, sketch the solid, draw a representative rectangle, write a representative element and label the sketch completely.

$x = \sqrt{y-2}+1$, $y = 3$, $x = 3$; about $y = 1$

Intersection between $x=3$ and $x=\sqrt{y-2}+1$

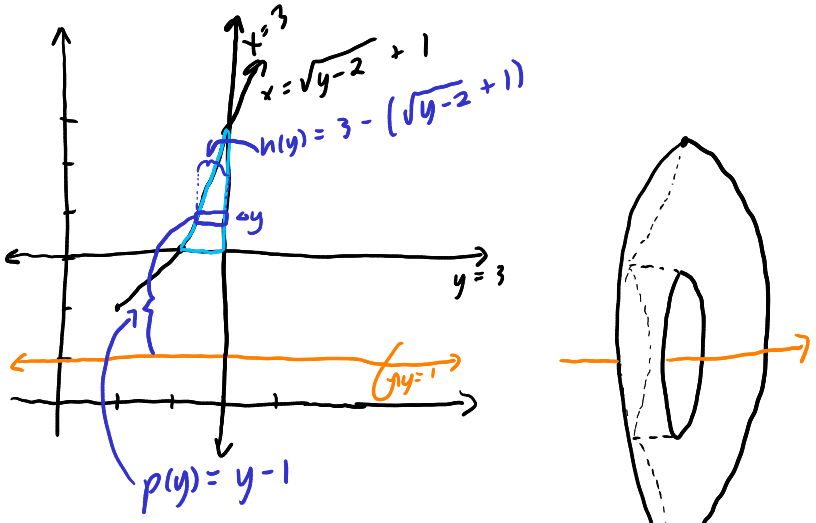
$3 = \sqrt{y-2}+1$

$2 = \sqrt{y-2}$

$4 = y-2$

$6 = y$

a. *(5 marks)* Using the shell method.



$\Delta V = 2\pi p(y) h(y) \Delta y$

$= 2\pi (y-1)(3-(\sqrt{y-2}+1))\Delta y$

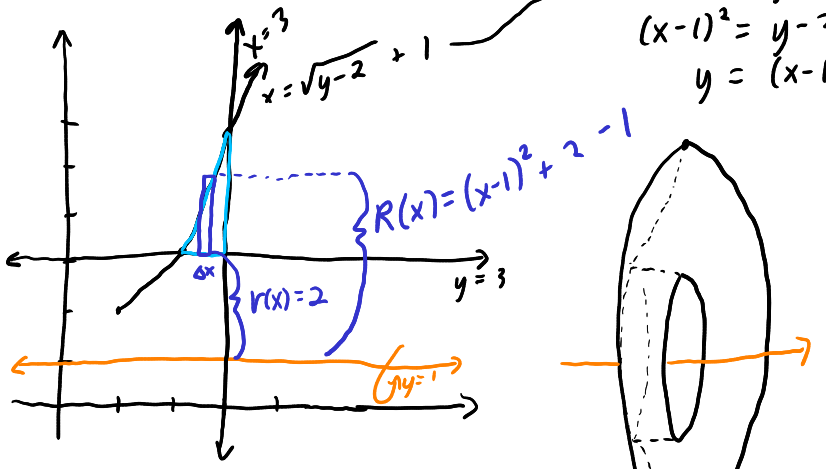
$V = \int_3^6 2\pi (y-1)(3-(\sqrt{y-2}+1))\,dy$

b. *(5 marks)* Using the washer method.

$x-1 = \sqrt{y-2}$

$(x-1)^2 = y-2$

$y = (x-1)^2+2$



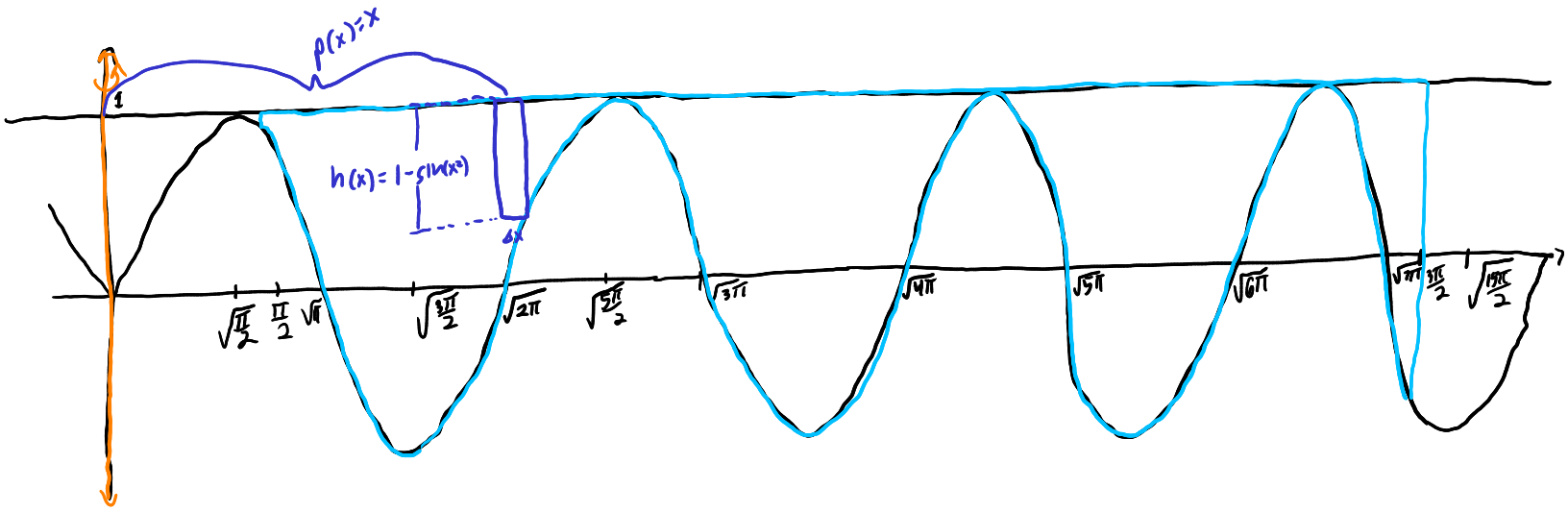
$\Delta V = \pi [(R(x))^2 - (r(x))^2]\Delta x$

$= \pi [((x-1)^2+1)^2 - 2^2]\Delta x$

$V = \int_2^3 \pi [((x-1)^2+1)^2 - 2^2]\,dx$

Question 2. *(5 marks)* **Find** the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis using the specified method. Sketch the region, draw a representative rectangle, write a representative element and label the sketch completely.

$y = \sin(x^2)$, $y = 1$, $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$, about the y-axis



$$\Delta V = 2\pi\, p(x)\, h(x)\, \Delta x$$
$$= 2\pi \times (1 - \sin x^2)\, \Delta x$$

$$V = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2\pi \times (1 - \sin x^2)\, dx$$

$$= 2\pi \int_{\pi/2}^{\frac{3\pi}{2}} x - x \sin x^2\, dx$$

$$= 2\pi \left[\frac{1}{2}x^2 + \frac{\cos x^2}{2} \right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$= \pi \left[\left(\frac{3\pi}{2}\right)^2 - \left(\frac{\pi}{2}\right)^2 + \cos\left(\frac{3\pi}{2}\right)^2 - \cos\left(\frac{\pi}{2}\right)^2 \right]$$