

Question 1.

a. (2 marks) Show that $0 < \frac{(n!)^2}{(2n)!} < 1$ when $n \geq 1$.

b. (4 marks) Use part a. (whether or not you have shown it) to determine if the following sequence converges or diverges. If it converges, find its limit.

$$a_n = \frac{(n!)^2}{(2n+1)!}$$

Question 2. (3 marks) Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

$$\left\{ \frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \dots \right\}$$

Question 3. (5 marks) Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = n \int_0^{1/n} \arccos(t) dt$$