Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S3: Winter 2023: **Quiz 11** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.

a. *(2 marks)* Show that $0 < \frac{(n!)^2}{(2n)!} < 1$ when $n \geq 1$.

$0 < \frac{(n!)^2}{(2n)!}$ since factorials are the product of positive integers

$$\frac{(n!)^2}{(2n)!} = \frac{\cancel{(1\cdot 2\cdot 3 \cdots n)}(1\cdot 2\cdot 3\cdots n)}{\cancel{1\cdot 2\cdot 3 \cdots n}(n+1)(n+2)(n+3)\cdots 2n} = \underbrace{\frac{1}{n+1}}_{\leq 1}\cdot\underbrace{\frac{2}{n+2}}_{\leq 1}\cdot\underbrace{\frac{3}{n+3}}_{\leq 1}\cdots\underbrace{\frac{n}{2n}}_{\leq 1} < 1$$

b. *(4 marks)* Use part a. (whether or not you have shown it) to determine if the following sequence converges or diverges. If it converges, find its limit.

$$a_n = \frac{(n!)^2}{(2n+1)!}$$

$$0 < \frac{(n!)^2}{(2n)!} < 1$$

$$0\,\frac{1}{2n+1} < \frac{(n!)^2}{(2n)!}\,\frac{1}{2n+1} < 1\cdot\frac{1}{2n+1}$$

$$c_n = 0 < a_n < \frac{1}{2n+1} = b_n$$

Since $\lim\limits_{n\to\infty} c_n = \lim\limits_{n\to\infty} b_n = 0$ then by the squeeze thm. $a_n \to 0$ as $n \to \infty$

Question 2. *(3 marks)* Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

$$\left\{\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \ldots\right\} \qquad a_n = (-1)^{n+1}\frac{n^2}{n+1}$$

Question 3. *(5 marks)* Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = n\int_0^{1/n} \arccos(t)\, dt$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} n \int_0^{1/n} \arccos(t)dt \qquad \text{I.F. } \infty\cdot 0$$

$$= \lim_{x\to\infty} x \int_0^{1/x} \arccos(t)\,dt$$

$$= \lim_{x\to\infty} \frac{\int_0^{1/x} \arccos(t)\,dt}{1/x} \qquad \text{I.F. } \frac{0}{0}$$

$$\overset{H}{=} \lim_{x\to\infty} \frac{\arccos\left(\frac{1}{x}\right)\cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}}$$

$$= \arccos(0)$$

$$= \frac{\pi}{2}$$