Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S3: Winter 2023: **Quiz 12** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=2}^{\infty}\left[\underbrace{\frac{1}{\pi^n}}_{a_n}+\underbrace{e^{1/n}-e^{1/(n+1)}}_{b_n}\right]$$

Lets look at the series $\sum a_n$ and $\sum b_n$ independently

$$\sum_{n=2}^{\infty} a_n = \sum_{n=2}^{\infty} \frac{1}{\pi^n} = \sum_{n=2}^{\infty} \left(\frac{1}{\pi}\right)^n = \sum_{n=2}^{\infty} \left(\frac{1}{\pi}\right)^{n-2+2} = \sum_{n=2}^{\infty} \left(\frac{1}{\pi}\right)^2 \left(\frac{1}{\pi}\right)^{n-2} \quad \text{conv. since } |r|<1$$

$$= \frac{(1/\pi)^2}{1-\frac{1}{\pi}}$$

$$= \frac{1}{\pi(\pi-1)}$$

$$S_n = b_2 + b_3 + b_4 + \cdots + b_{n-2} + b_{n-1} + b_n$$

$$= [e^{1/2} - \cancel{e^{1/3}}] + [\cancel{e^{1/3}} - \cancel{e^{1/4}}] + [\cancel{e^{1/4}} - e^{1/5}] + \cdots + [e^{1/(n-2)} - \cancel{e^{1/(n-1)}}] + [\cancel{e^{1/(n-1)}} - \cancel{e^{1/n}}] + [\cancel{e^{1/n}} - e^{\frac{1}{n+1}}]$$

$$= e^{1/2} - e^{1/(n+1)}$$

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} e^{1/2} - e^{\frac{1}{n+1}} = e^{1/2} - e^0 = e^{1/2} - 1$$

$$\therefore \sum_{n=2}^{\infty} b_n = e^{1/2} - 1$$

$$\therefore \sum_{n=2}^{\infty} [a_n + b_n] = \sum_{n=2}^{\infty} a_n + \sum_{n=2}^{\infty} b_n = \frac{1}{\pi(\pi-1)} + e^{1/2} - 1$$

Question 2. *(5 marks)* If the nth partial sum of a series $\sum_{n=1}^{\infty} a_n$ is $S_n = 3 - n2^{-n}$, find a_n and $\sum_{n=1}^{\infty} a_n$.

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} (3 - n2^{-n}) = 3 - \lim_{x\to\infty} \frac{x}{2^x} \quad \text{i.f. } \frac{\infty}{\infty}$$

$$\overset{H}{=} 3 - \lim_{x\to\infty} \frac{1}{\cancel{2^x}(\ln 2)} \to 0$$

$$= 3$$

$$\therefore \sum a_n = 3$$

① $S_n = a_1 + a_2 + \cdots + a_{n-1} + a_n$

② $S_{n-1} = a_1 + a_2 + \cdots + a_{n-1}$

① - ② $\quad S_n - S_{n-1} = a_n$

$$\therefore a_n = S_n - S_{n-1}$$

$$= 3 - n2^{-n} - (3 - (n-1)2^{-(n-1)})$$

$$= -n2^{-n} + (n-1)2^{-(n-1)}$$

Question 3. *(5 marks)* Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=2}^{\infty} \underbrace{\int_0^{\pi/2-1/n} \sin^2 x\,dx}_{a_n}$$

Lets look at

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \int_0^{\pi/2 - 1/n} \sin^2 x\,dx$$

$$= \int_0^{\pi/2} \sin^2 x\,dx$$

$$= \int_0^{\pi/2} \frac{1-\cos 2x}{2}\,dx$$

$$= \left[\frac{1}{2}x - \frac{\sin 2x}{4}\right]_0^{\pi/2}$$

$$= \frac{\pi}{4} - \frac{\sin 2\left(\frac{\pi}{2}\right)}{4} = \frac{\pi}{4} \neq 0$$

$\therefore$ by the n^{th} term div. test the series is divergent.