**Dawson College**: **Calculus II** (SCIENCE): 201-NYB-05-S3: Winter 2023: **Quiz 13** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.** *(5 marks)* Determine whether the series is convergent or divergent.

$$\sum_{n=2}^{\infty} \underbrace{\cos\left(\frac{1}{n^2}\right)}_{a_n}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \cos\left(\frac{1}{n^2}\right) = \cos(0) = 1 \neq 0$$

$\therefore$ the series is divergent by the $n^{th}$ term divergence test.

**Question 2.** *(5 marks)* Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{n-\arcsin\left(\frac{1}{n}\right)}{n2^n}$$

Let $a_n = \dfrac{n-\arcsin\left(\frac{1}{n}\right)}{n\,2^n} \leq \dfrac{n}{n\,2^n} = \dfrac{1}{2^n} = \left(\dfrac{1}{2}\right)^n = b_n$

$0 \leq a_n$

$\sum a_n$ converges by the comparison test since $0 \leq a_n \leq b_n$

and $\sum b_n$ is convergent because its a geometric series where

$|r| = \frac{1}{2} < 1$.

**Question 3.** *(5 marks)* Determine whether the series is convergent or divergent.

$$\sum_{n=10}^{\infty} \frac{1}{n\ln n\sqrt{(\ln n)^2 - 1}}$$

Let $a_n = f(n)$ where $f(x) = \dfrac{1}{x\ln x\sqrt{(\ln x)^2 - 1}}$

- $f(x) > 0$ on $[10, \infty)$
- $f(x)$ is continuous on $[10, \infty)$
- $f'(x) = \dfrac{-1}{\left(x\ln x\sqrt{(\ln x)^2 - 1}\right)^2}\left[\ln x\sqrt{(\ln x)^2 - 1} + x\frac{1}{x}\sqrt{(\ln x)^2-1} + x\ln x \frac{1}{\sqrt{(\ln x)^2 - 1}}\frac{2\ln x}{x}\right]$

$< 0$ on $[10, \infty)$

So lets apply the integral test

$$\int_{10}^{\infty} \frac{1}{x\ln x\sqrt{(\ln x)^2 - 1}}\,dx = \lim_{b\to\infty}\int_{10}^{b} \frac{1}{x\ln x\sqrt{(\ln x)^2 - 1}}\,dx$$

$u = \ln x$

$du = \frac{1}{x}dx$

$u(b) = \ln b$

$u(10) = \ln 10$

$$= \lim_{b\to\infty}\int_{\ln 10}^{\ln b} \frac{1}{u\sqrt{u^2 - 1}}\,du$$

$$= \lim_{b\to\infty}\Big[\operatorname{arcsec} u\Big]_{\ln 10}^{\ln b}$$

$$= \lim_{b\to\infty} \operatorname{arcsec}\ln b - \operatorname{arcsec}\ln 10$$

$$= \frac{\pi}{2} - \operatorname{arcsec}(\ln(10))$$

$\therefore$ the improper integral converges

$\therefore$ by the integral test the series $\sum a_n$ converges since the improper integral converges.

**Bonus Question** *(2 marks)*

A Calculus II student is hired as a manager at David Hilbert's Hotel which is a very large hotel, in fact, it has infinitely many rooms numbered 1, 2, 3, …. The hotel is very popular and every room is occupied one night.

That night a new guest arrives.

-Is there any free room?

-No, the former Calculus II student said.

-Oh, what a pity, the guest said and started to walk away.

-But wait, you can still get a room.

How can the former Calculus II student find a room for the guest?