Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S3: Winter 2023: **Quiz 15** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Find the radius of convergence and interval of convergence of the following series:

$$\sum_{n=1}^{\infty} \underbrace{\frac{(2x-1)^n}{5^n\sqrt{n}}}_{a_n(x)}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}(x)}{a_n(x)}\right| = \lim_{n\to\infty}\left|\frac{\frac{(2x-1)^{n+1}}{5^{n+1}\sqrt{n+1}}}{\frac{(2x-1)^n}{5^n\sqrt{n}}}\right| = \lim_{n\to\infty}\left|\frac{(2x-1)^{n+1}}{5^{n+1}\sqrt{n+1}}\cdot\frac{5^n\sqrt{n}}{(2x-1)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{\cancel{(2x-1)^n}(2x-1)\,\cancel{5^n}\sqrt{n}}{\cancel{5^n}5\sqrt{n+1}\,\cancel{(2x-1)^n}}\right|$$

$$= |2x-1|\lim_{n\to\infty}\frac{\sqrt{n}}{5\sqrt{n+1}}$$

$$= |2x-1|\frac{1}{5} < 1 \quad \text{ratio test, series converges if } < 1$$

$$2|x-1/2|\frac{1}{5} < 1$$

$$|x-\tfrac{1}{2}| < \frac{5}{2} = R$$

$$-\frac{5}{2} < x - \frac{1}{2} < \frac{5}{2}$$

$$-2 < x < 3$$

Lets test the endpoints.

Let $x=-2$

$$\Rightarrow \sum_{n=1}^{\infty}\frac{(2(-2)-1)^n}{5^n\sqrt{n}} = \sum_{n=1}^{\infty}\frac{(-5)^n}{5^n\sqrt{n}} = \sum_{n=1}^{\infty}\frac{(-1)^n\cancel{5^n}}{\cancel{5^n}\sqrt{n}} = \sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}}$$

Let $b_n = \frac{1}{\sqrt{n}}$

1) $\lim_{n\to\infty} b_n = 0$

2) $b_{n+1} < b_n$ since $\frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}}$

$\therefore$ converges at $x=-2$ by the alternating series test

Let $x=3$

$$\Rightarrow \sum_{n=1}^{\infty}\frac{(2(3)-1)^n}{5^n\sqrt{n}} = \sum_{n=1}^{\infty}\frac{5^n}{5^n\sqrt{n}} = \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$$ diverges since p-series where $p < \frac{1}{2}$.

$\therefore$ interval of convergence is $[-2, 3)$

Question 2. *(5 marks)* Find the Taylor series for $f(x) = \sqrt{x}$ centered at $a = 16$.

$f(x) = \sqrt{x}$ $\qquad$ $f(16) = \sqrt{16} = 4$

$f'(x) = \dfrac{1}{2x^{1/2}}$ $\qquad$ $f'(16) = \dfrac{1}{2\,16^{1/2}} = \dfrac{1}{2 \cdot 4}$

$f''(x) = \dfrac{-1}{2^2 x^{3/2}}$ $\qquad$ $f''(16) = \dfrac{-1}{2^2 16^{3/2}} = \dfrac{-1}{2^2 \cdot 4^3}$

$f'''(x) = \dfrac{1 \cdot 3}{2^3 x^{5/2}}$ $\qquad$ $f'''(16) = \dfrac{1 \cdot 3}{2^3 16^{5/2}} = \dfrac{1 \cdot 3}{2^3 4^5}$

$f^{(4)}(x) = \dfrac{-1 \cdot 3 \cdot 5}{2^4 x^{7/2}}$ $\qquad$ $f^{(4)}(16) = \dfrac{1 \cdot 3 \cdot 5}{2^4 16^{7/2}} = \dfrac{-1 \cdot 3 \cdot 5}{2^4 4^7}$

$\vdots$ $\qquad$ $\vdots$

$f^{(n)}(x) = (-1)^{n+1} \dfrac{1 \cdot 3 \cdot 5 \cdots (2n-3)}{2^n x^{(2n-1)/2}}$ $\qquad$ $f^{(n)}(16) = \cdots = (-1)^{n+1} \dfrac{1 \cdot 3 \cdot 5 \cdots (2n-3)}{2^n 4^{2n-1}}$

for $n \geq 2$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = \frac{f^{(0)}(16)}{0!}(x-16)^0 + \frac{f^{(1)}(16)}{1!}(x-16)^1 + \sum_{n=2}^{\infty} \frac{f^{(n)}(16)}{n!}(x-16)^2$$

$$= 4 + \frac{1}{8}(x-16) + \sum_{n=2}^{\infty} \frac{(-1)^{n+1} 1 \cdot 3 \cdot 5 \cdots (2n-3)}{2^n 4^{2n-1} n!}(x-16)^n$$