Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S3: Winter 2023: **Quiz 3** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks)* Determine and sketch the region whose area is equal to

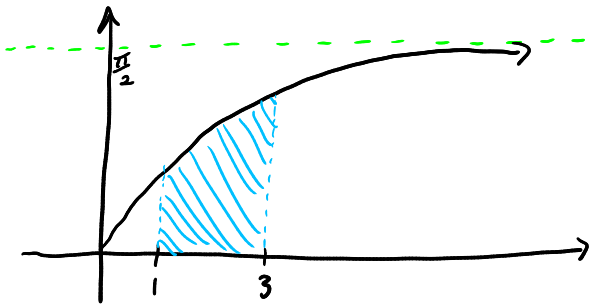
$$\lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n} \arctan\left(1+\frac{2i}{n}\right) = \lim_{n\to\infty} \sum_{i=1}^{n} \arctan\left(1+\frac{2i}{n}\right)\frac{2}{n}$$

$f(x) = \arctan x$

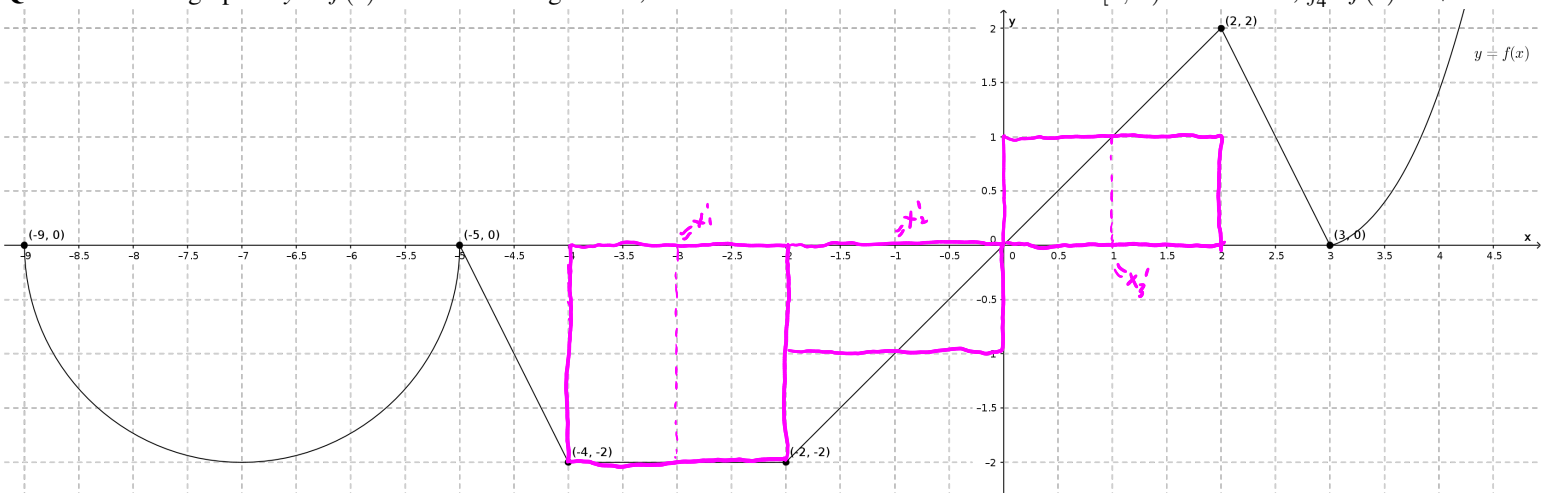
$\Delta x = \frac{2}{n} = \frac{b-a}{n}$

$x_i = 1 + \frac{2i}{n} = a + i\Delta x$ so $a = 1$, $b = 3$

Do not evaluate the limit.



Question 2. The graph of $y = f(x)$ consists of straight lines, one semicircle and a curve on the interval $[3,\infty)$. In addition, $\int_4^3 9f(x)\,dx + 4 = 0$.



a. *(3 marks)* Find an approximation of the definite integral of $f(x)$ on the interval $[-4,2]$, using the midpoints as sample points and 3 approximating rectangles. Draw the approximating rectangles. $\Delta x = \frac{b-a}{n} = \frac{2-(-4)}{3} = \frac{6}{3} = 2$

b. *(5 marks)* Evaluate $\int_{-1}^{4} f(x)\,dx$.

a) $\int_{-4}^{2} f(x)dx \approx f(-3)\Delta x + f(-1)\Delta x + f(1)\Delta x = (-2)(2) + (-1)(2) + (1)(2) = -4$

b) $\int_4^3 9f(x)dx + 4 = 0$

$\int_4^3 9f(x)dx = -4$

$9\int_4^3 f(x)dx = -4$

$-9\int_3^4 f(x)dx = -4$

$\int_3^4 f(x)dx = \frac{4}{9}$

$$\int_{-1}^{4} f(x)dx = \int_{-1}^{0} f(x)dx + \int_{0}^{3} f(x)dx + \int_{3}^{4} f(x)dx$$

$$= \frac{(-1)(1)}{2} + \frac{2(3)}{2} + \frac{4}{9}$$

$$= -\frac{1}{2} + 3 + \frac{4}{9}$$

$$= \frac{5}{2} + \frac{4}{9}$$

$$= \frac{45+8}{18}$$

$$= \frac{53}{18}$$