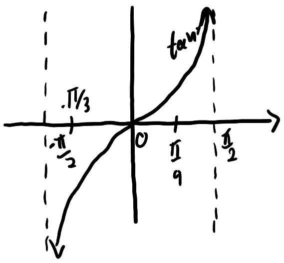
Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S3: Winter 2023: **Quiz 4** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Find the average value of $f(x) = |\tan x|$ on the interval $[-\pi/3, \pi/4]$.



avg. val. of $f(x)$ on $[-\frac{\pi}{3}, \frac{\pi}{4}]$ $= \frac{1}{b-a}\int_a^b f(x)dx$

$$= \frac{1}{\pi/4 - (-\pi/3)} \int_{-\frac{\pi}{3}}^{\pi/4} |\tan x| dx$$

$$= \frac{1}{\frac{3\pi}{12} + \frac{4\pi}{12}} \left[\int_{-\frac{\pi}{3}}^{0} |\tan x| dx + \int_0^{\pi/4} |\tan x| dx \right]$$

$$= \frac{12}{7\pi} \left[\int_{-\frac{\pi}{3}}^{0} -\tan x \, dx + \int_0^{\pi/4} \tan x \, dx \right]$$

$$= \frac{12}{7\pi} \left[\left[+\ln|\cos x| \right]_{-\frac{\pi}{3}}^{0} + \left[-\ln|\cos x| \right]_0^{\pi/4} \right]$$

$$= \frac{12}{7\pi} \left[\underbrace{\ln|\cos 0|}_{0} - \ln|\cos -\tfrac{\pi}{3}| \right] + -\ln|\cos \tfrac{\pi}{4}| - \underbrace{[-\ln|\cos 0|]}_{0}$$

$$= \frac{12}{7\pi} \left[-\ln \frac{1}{2} - \ln \frac{1}{\sqrt{2}} \right] = \frac{12}{7\pi} \ln 2\sqrt{2} = \ln \left(2\sqrt{2}\right)^{12/7\pi}$$

Question 2. *(5 marks)* Find the derivative of the function and simplify.

$$\frac{d}{dx}\left[\int_x^{\arctan x} \tan^{2023}\theta \, d\theta\right]$$

Let $h(x) = \int_x^{\arctan x} \tan^{2023}\theta \, d\theta$

$$= \int_x^0 \tan^{2023}\theta \, d\theta + \int_0^{\arctan x} \tan^{2023}\theta \, d\theta$$

$$= -\int_0^x \tan^{2023}\theta \, d\theta + \int_0^{\arctan x} \tan^{2023}\theta \, d\theta$$

$$= -f(x) + f(g(x))$$

where $f(x) = \int_0^x \tan^{2023}\theta \, d\theta$ $\quad f'(x) = \tan^{2023} x$ by 2^{nd} FTC

$g(x) = \arctan x$ $\quad g'(x) = \frac{1}{x^2+1}$

$$= \frac{d}{dx}[h(x)]$$

$$= f'(x) + f'(g(x))g'(x)$$

$$= -\tan^{2023} x + \frac{\tan^{2023} \arctan x}{x^2+1}$$

$$= -\tan^{2023} x + \frac{x^{2023}}{x^2+1}$$