**Dawson College**: **Calculus II** (SCIENCE): 201-NYB-05-S3: Winter 2023: **Quiz 5** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.** *(4 marks)* Evaluate the definite integral.

$$\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx = \int_1^9 \frac{\frac{u-1}{2}}{\sqrt{u}}\,\frac{du}{2}$$

$u = 1+2x$
$du = 2dx$
$\frac{du}{2} = dx$
$u(4) = 1+2(4) = 9$
$u(0) = 1+2(0) = 1$
$u-1 = 2x$
$\frac{u-1}{2} = x$

$$= \frac{1}{4}\int_1^9 (u-1)u^{-1/2}\,du$$

$$= \frac{1}{4}\int_1^9 u^{1/2} - u^{-1/2}\,du$$

$$= \frac{1}{4}\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]_1^9$$

$$= \frac{1}{4}\left[\left[\frac{2}{3}9^{3/2} - 2\,9^{1/2}\right] - \left[\frac{2}{3}1^{3/2} - 2\,1^{1/2}\right]\right]$$

$$= \frac{1}{4}\left[\left[\frac{2}{3}27 - 6\right] - \left[\frac{2}{3} - 2\right]\right]$$

$$= \frac{1}{4}\left[18 - 6 - \frac{2}{3} + 2\right]$$

$$= \frac{1}{4}\left[40/3\right] = 10/3$$

**Question 2.** *(3 marks)* If $a$ and $b$ are positive numbers, show that

$$\int_0^1 x^a(1-x)^b\,dx = \int_0^1 x^b(1-x)^a\,dx$$

$$\int_0^1 x^a(1-x)^b\,dx = \int_1^0 (1-u)^a u^b(-du)$$

$u = 1-x$
$du = -dx$
$-du = dx$
$u(1) = 1-1 = 0$
$u(0) = 1-0 = 1$
$x = 1-u$

$$= -\int_1^0 (1-u)^a u^b\,du$$

$$= \int_0^1 (1-u)^a u^b\,du$$

$$= \int_0^1 x^b(1-x)^a\,dx$$

**Question 3.** *(4 marks)* Evaluate the integral

$$I = \int \cos(\ln x)\,dx = uv - \int v\,du$$

$u = \cos(\ln x)$ $du = -\sin(\ln x)\frac{1}{x}dx$
$v = x$ $dv = dx$

$$I = x\cos(\ln x) - \int x(-\sin(\ln x))\frac{1}{x}\,dx$$

$$I = x\cos(\ln x) + \int \sin(\ln x)\,dx$$

$u = \sin(\ln x)$ $du = \cos(\ln x)\frac{1}{x}dx$
$v = x$ $dv = dx$

$$I = x\cos(\ln x) + uv - \int v\,du$$

$$I = x\cos(\ln x) + x\sin(\ln x) - \int x\cos(\ln x)\cancel{\tfrac{1}{x}}\,dx$$

$$I = x\cos(\ln x) + x\sin(\ln x) - I + C$$

$$2I = x\cos(\ln x) + x\sin(\ln x) + C$$

$$I = \frac{1}{2}\left[x\cos(\ln x) + x\sin(\ln x)\right] + C$$

**Question 4.** *(3 marks)* If $f(0) = g(0) = 0$ and $f''$ and $g''$ are continuous, show that

$$\int_0^a f(x)g''(x)\,dx = f(a)g'(a) - f'(a)g(a) + \int_0^a f''(x)g(x)\,dx$$

$$\int_0^a f(x)g''(x)\,dx = [uv]_0^a - \int_0^a v\,du$$

$u = f(x)$ $du = f'(x)dx$
$v = g'(x)$ $dv = g''(x)dx$

$$= \left[f(x)g'(x)\right]_0^a - \int_0^a f'(x)g'(x)\,dx$$

$u = f'(x)$ $du = f''(x)dx$
$v = g(x)$ $dv = g'(x)dx$

$$= f(a)g'(a) - \underbrace{f(0)g'(0)}_{0} - \left[[uv]_0^a - \int_0^a v\,du\right]$$

$$= f(a)g'(a) - \left[\left[f'(x)g(x)\right]_0^a - \int_0^a g(x)f''(x)\,dx\right]$$

$$= f(a)g'(a) - f'(a)g(a) + \underbrace{f'(0)g(0)}_{0} + \int_0^a g(x)f''(x)\,dx = f(a)g'(a) - f'(a)g(a) + \int_0^a f''(x)g(x)\,dx$$