Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S3: Winter 2023: **Quiz 6** name: Y.Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Evaluate the integral

$$\int_{\pi/4}^{\pi/2} \cot^3 x \, dx = \int_{\pi/4}^{\pi/2} \frac{\cos^3 x}{\sin^3 x} dx$$

$$= \int_{\pi/4}^{\pi/2} \frac{\cos^2 x}{\sin^3 x} \cos x \, dx$$

$$= \int_{\pi/4}^{\pi/2} \frac{1-\sin^2 x}{\sin^3 x} \cos x \, dx \qquad \begin{array}{l} u = \sin x \\ du = \cos x \, dx \end{array} \qquad \begin{array}{l} u(\pi/2) = \sin \pi/2 = 1 \\ u(\pi/4) = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} \end{array}$$

$$= \int_{1/\sqrt{2}}^{1} \frac{1-u^2}{u^3} du$$

$$= \int_{1/\sqrt{2}}^{1} u^{-3} - \frac{1}{u} du$$

$$= \left[\frac{u^{-2}}{-2} - \ln|u| \right]_{1/\sqrt{2}}^{1}$$

$$= \left[\frac{1^{-2}}{-2} - \ln|1| \right] - \left[\frac{(1/\sqrt{2})^{-2}}{-2} - \ln\left|\frac{1}{\sqrt{2}}\right| \right]$$

$$= \frac{-1}{2} + 1 + \ln\left(\frac{1}{\sqrt{2}}\right)$$

$$= 1/2 - \ln\sqrt{2}$$

Question 2. *(5 marks)* Evaluate the integral

$$\int \frac{x^2}{(3+4x-4x^2)^{3/2}}\,dx = \int \frac{x^2}{(4-(2x-1)^2)^{3/2}}\,dx = \int \frac{\left(\frac{u+1}{2}\right)^2}{(4-u^2)^{3/2}}\,\frac{du}{2}$$

$$-4x^2+4x+3$$
$$= -4\left[x^2 - x - \tfrac{3}{4}\right]$$
$$= -4\left[x^2 - x + \tfrac{1}{4} - \tfrac{1}{4} - \tfrac{3}{4}\right]$$
$$= -4\left[(x-\tfrac{1}{2})^2 - 1\right]$$
$$= 4 - 4(x-\tfrac{1}{2})^2$$
$$= 4 - (2x-1)^2$$

$u = 2x-1$

$du = 2dx$

$x = \frac{u+1}{2}$

$u = 2\sin\theta$

$du = 2\cos\theta\, d\theta$

$$= \frac{1}{8}\int \frac{\overbrace{u^2+2u+1}^{f(u)}}{(4-u^2)^{3/2}}\,du$$

$$= \frac{1}{8}\int \frac{(2\sin\theta)^2 + 4\sin\theta + 1}{(4-4\sin^2\theta)^{3/2}}\, 2\cos\theta\, d\theta$$

$$= \frac{2}{8}\int \frac{4\sin^2\theta + 4\sin\theta + 1}{(4\cos^2\theta)^{3/2}}\cos\theta\, d\theta$$

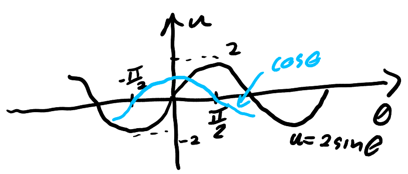
$$= \frac{1}{4}\int \frac{4\sin^2\theta + 4\sin\theta + 1}{2^3(|\cos\theta|)^3}\cos\theta\, d\theta$$

$$= \frac{1}{32}\int \frac{4\sin^2\theta + 4\sin\theta + 1}{\cancel{\cos^3\theta}\,\cos^2\theta}\,\cancel{\cos\theta}\, d\theta$$

since $\cos\theta > 0$ on $(-\frac{\pi}{2}, \frac{\pi}{2})$

Domain of $f(u)$ $(-2, 2)$

$\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$



$$= \frac{1}{32}\int 4\tan^2\theta + 4\frac{\sin\theta}{\cos^2\theta} + \sec^2\theta\, d\theta$$

$$= \frac{1}{32}\left[\int 4(\sec^2\theta - 1) + \sec^2\theta\, d\theta + \int \frac{4\sin\theta}{\cos^2\theta}\, d\theta\right]$$

$v = \cos\theta$

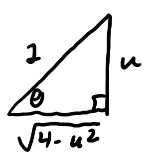
$dv = -\sin\theta\, d\theta$

$$= \frac{1}{32}\left[\int 5\sec^2\theta - 4\, d\theta + \int \frac{-4}{v^2}\, dv\right]$$

$$= \frac{1}{32}\left[5\tan\theta - 4\theta + 4v^{-1}\right] + C$$

$u = 2\sin\theta$

$\frac{u}{2} = \sin\theta \Rightarrow \arcsin\left(\frac{u}{2}\right) = \theta$



$$= \frac{1}{32}\left[5\frac{u}{\sqrt{4-u^2}} - 4\arcsin\left(\frac{u}{2}\right) + 4\sec\theta\right] + C$$

$$= \frac{1}{32}\left[5\frac{(2x-1)}{\sqrt{4-(2x-1)^2}} - 4\arcsin\left(\frac{2x-1}{2}\right) + 4\cdot\frac{2}{\sqrt{4-(2x-1)^2}}\right] + C$$