Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S3: Winter 2023: **Quiz 7** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Evaluate the integral

$$\int x^2 \arctan x \, dx = uv - \int v\,du$$

$$\boxed{\begin{array}{ll} u = \arctan x & du = \dfrac{1}{x^2+1}\,dx \\ v = \dfrac{x^3}{3} & dv = x^2\,dx \end{array}}$$

$$= \frac{x^3}{3}\arctan x - \frac{1}{3}\int \frac{x^3}{x^2+1}\,dx$$

$$= \frac{x^3}{3}\arctan x - \frac{1}{3}\int x - \frac{x}{x^2+1}\,dx$$

$$\begin{array}{r|l} & x \\ \hline x^2+0x+1 & x^3+0x^2+0x+0 \\ & -(x^3 \quad +x) \\ \hline & -x \end{array}$$

$$= \frac{x^3}{3}\arctan x - \frac{1}{3}\left[\frac{x^2}{2} - \int \frac{x}{x^2+1}\,dx\right]$$

$$u = x^2+1$$
$$du = 2x\,dx$$
$$\frac{du}{2} = x\,dx$$

$$= \frac{x^3}{3}\arctan x - \frac{x^2}{6} + \frac{1}{3}\int \frac{1}{u}\,\frac{du}{2}$$

$$= \frac{x^3}{3}\arctan x - \frac{x^2}{6} + \frac{1}{6}\ln|u| + C$$

$$= \frac{x^3}{3}\arctan x - \frac{x^2}{6} + \frac{1}{6}\ln(x^2+1) + C$$

Question 2. *(5 marks)* Evaluate the integral

$$\int \frac{1}{x^3-1}\,dx$$

Possible rational roots of x^3-1 are ± 1. Since $(1)^3-1=0$ we have that $x=1$ is a root of x^3-1 and $x-1$ is a factor of x^3-1.

$\therefore$

$$\begin{array}{r|l} & x^2+x+1 \\ \hline x-1 & x^3+0x^2+0x-1 \\ & -(x^3-x^2) \\ \hline & \quad x^2+0x \\ & \quad -(x^2-x) \\ \hline & \qquad x-1 \\ & \qquad -(x-1) \\ \hline & \qquad\quad 0 \end{array}$$

$$= \int \frac{1}{(x-1)(x^2+x+1)}\,dx = \int \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}\,dx = \int \frac{1/3}{x-1} + \frac{-1/3x - 2/3}{x^2+x+1}\,dx$$

$$\frac{1}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

$$1 = A(x^2+x+1) + (Bx+C)(x-1)$$

Let $x=1$

$$1 = A(1^2+1+1) + (B(1)+C)(1-1)$$

$$1/3 = A$$

Let $x=0$

$$1 = \tfrac{1}{3}(0^2+0+1) + (B(0)+C)(0-1)$$

$$1 = 1/3 - C$$

$$C = -2/3$$

Let $x=-1$

$$1 = 1/3((-1)^2-1+1) + (B(-1)+(-2))(-1-1)$$

$$1 = 1/3 + (-B-2/3)(-2)$$

$$1 = 1/3 + 2B + 4/3$$

$$2B = -2/3$$

$$B = -1/3$$

$$x^2+x+1 = x^2+x+\tfrac{1}{4}-\tfrac{1}{4}+1$$

$$= (x+\tfrac{1}{2})^2+\tfrac{3}{4}$$

$$= \frac{1}{3}\ln|x-1| + \frac{1}{6}\int \frac{-2x-4}{x^2+x+1}\,dx$$

$$= \frac{1}{3}\ln|x-1| + \frac{1}{6}\left[\int \frac{-2x-1}{x^2+x+1} - \frac{3}{x^2+x+1}\,dx\right]$$

$$u = x^2+x-1$$

$$du = (2x+1)dx$$

$$-du = (-2x-1)dx$$

$$= \frac{1}{3}\ln|x-1| + \frac{1}{6}\int \frac{-du}{u} - \frac{1}{2}\int \frac{1}{x^2+x+1}\,dx$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln|u| - \frac{1}{2}\int \frac{1}{x^2+x+1}\,dx$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln|x^2+x+1| - \frac{1}{2}\int \frac{1}{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln|x^2+x+1| - \frac{1}{2}\frac{2}{\sqrt{3}}\arctan\left(\frac{x+1/2}{\frac{\sqrt{3}}{2}}\right) + C$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln|x^2+x+1| - \frac{1}{\sqrt{3}}\arctan\left(\frac{2x+1}{\sqrt{3}}\right) + C$$