Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S3: Winter 2023: **Quiz 8** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Determine whether the integral is convergent or divergent. Evaluate if it is convergent.

$$\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx = \lim_{a\to 0^+} \int_a^1 \frac{\ln x}{\sqrt{x}}\,dx$$

$u = \ln x$ $\quad du = \frac{1}{x}dx$

$v = 2\sqrt{x}$ $\quad dv = \frac{1}{\sqrt{x}}dx$

$$= \lim_{a\to 0^+}\left[\left[uv\right]_a^1 - \int_a^1 v\,du\right]$$

$$= \lim_{a\to 0^+}\left[\left[2\sqrt{x}\ln x\right]_a^1 - \int_a^1 \frac{2\sqrt{x}}{x}\,dx\right]$$

$$= \lim_{a\to 0^+}\left[\overbrace{2\sqrt{1}\ln 1}^{0} - 2\sqrt{a}\ln a - \left[4\sqrt{x}\right]_a^1\right]$$

$$= \lim_{a\to 0^+}\left[-2\sqrt{a}\ln a - 4\sqrt{1} + 4\sqrt{a}^{\to 0}\right]$$

$$= -4 - 2\lim_{a\to 0^+}\sqrt{a}\ln a \qquad \text{I.F. } 0\cdot(-\infty)$$

$$= -4 - 2\lim_{a\to 0^+}\frac{\ln a}{1/\sqrt{a}} \qquad \text{I.F. } \frac{-\infty}{\infty}$$

$$\overset{\hat{H}}{=} -4 - 2\lim_{a\to 0^+}\frac{1/a}{-\frac{1}{2}\frac{1}{a^{3/2}}}$$

$$= -4 + 4\lim_{a\to 0^+}\frac{a^{3/2}}{a}$$

$$= -4 + 4\lim_{a\to 0^+} a^{1/2} \to 0$$

$$= -4$$

Question 2. *(5 marks)* Find the limit.

$$\lim_{x\to 0}\csc x \int_{0.1}^{0.1-x}\frac{\ln(0.9+t)}{t}\,dt$$

$$= \lim_{x\to 0}\frac{\int_{0.1}^{0.1-x}\frac{\ln(0.9+t)}{t}\,dt}{\sin x} \qquad \text{I.F. } \frac{0}{0}$$

$$\overset{\hat{H}}{=} \lim_{x\to 0}\frac{\frac{\ln(1-x)}{0.1-x}(-1)}{\cos x}$$

$$= \lim_{x\to 0}\frac{-\ln(1-x)}{(0.1-x)\cos x}$$

$$= \frac{-\ln(1)}{0.1(\cos 0)} = 0$$

Question 3. *(5 marks)* Find the values of p for which the integral converges and evaluate the integral for those values of p.

$$\int_2^\infty \frac{(\text{arcsec}x)^p}{x\sqrt{x^2-1}}\,dx = \lim_{b\to\infty}\int_2^b \frac{(\text{arcsec}x)^p}{x\sqrt{x^2-1}}\,dx$$

$u = \text{arcsec } x$

$du = \frac{1}{x\sqrt{x^2-1}}\,dx$

$$= \lim_{b\to\infty}\int_{\pi/3}^{\text{arcsec} b} u^p\,dx$$

$u(b) = \text{arcsec} b$

$u(2) = \text{arcsec} 2 = \pi/3$

if $p=-1$ then

$$= \lim_{b\to\infty}\Big[\ln|u|\Big]_{\pi/3}^{\text{arcsec} b} = \lim_{b\to\infty}\Big[\ln \overset{\pi/2}{\text{arcsec} b} - \ln\frac{\pi}{3}\Big]$$

$$= \ln \pi/2 - \ln \pi/3 = \ln 3/2$$

$\therefore$ converges when $p=-1$

If $p \neq -1$ then

$$= \lim_{b\to\infty}\left[\frac{u^{p+1}}{p+1}\right]_{\pi/3}^{\text{arcsec} b} = \lim_{b\to\infty}\left[\frac{(\overset{\pi/2}{\text{arcsec} b})^{p+1}}{p+1} - \frac{(\pi/3)^{p+1}}{p+1}\right]$$

$$= \frac{(\pi/2)^{p+1}}{p+1} - \frac{(\pi/3)^{p+1}}{p+1}$$

$\therefore$ converges when $p \neq -1$

$\therefore$ converges $\forall p \in \mathbb{R}$