Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S3: Winter 2023: **Quiz 8** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Determine whether the integral is convergent or divergent. Evaluate if it is convergent.

$$\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx = \lim_{a\to 0^+} \int_a^1 \frac{\ln x}{\sqrt{x}}\,dx \qquad u = \ln x \qquad du = \frac{1}{x}dx$$

$$= \lim_{a\to 0^+} \left[\left[uv\right]_a^1 - \int_a^1 v\,du \right] \qquad v = 2\sqrt{x} \qquad dv = \frac{1}{\sqrt{x}}dx$$

$$= \lim_{a\to 0^+} \left[\left[2\sqrt{x}\ln x\right]_a^1 - \int_a^1 \frac{2\sqrt{x}}{x}dx \right]$$

$$= \lim_{a\to 0^+} \left[\overbrace{2\sqrt{1}\ln 1}^{0} - 2\sqrt{a}\ln a - \left[4\sqrt{x}\right]_a^1 \right]$$

$$= \lim_{a\to 0^+} \left[-2\sqrt{a}\ln a - 4\sqrt{1} + 4\underbrace{\sqrt{a}}_{\to 0} \right]$$

$$= -4 - 2\lim_{a\to 0^+} \sqrt{a}\ln a \qquad \text{I.F. } 0\cdot(-\infty)$$

$$= -4 - 2\lim_{a\to 0^+} \frac{\ln a}{1/\sqrt{a}} \qquad \text{I.F. } \frac{-\infty}{\infty}$$

$$\overset{\hat{H}}{=} -4 - 2\lim_{a\to 0^+} \frac{1/a}{-\frac{1}{2}\frac{1}{a^{3/2}}}$$

$$= -4 + 4\lim_{a\to 0^+} \frac{a^{3/2}}{a}$$

$$= -4 + 4\lim_{a\to 0^+} a^{1/2} \to 0$$

$$= -4$$

Question 2. *(5 marks)* Find the limit.

$$\lim_{x\to 0} \csc x \int_{0.1}^{0.1-x} \frac{\ln\ 0.9\quad t}{t}\,dt$$

$$= \lim_{x\to 0} \frac{\int_{0.1}^{0.1-x} \frac{\ln(0.9+t)}{t}\,dt}{\sin x} \qquad \text{I.F. } \frac{0}{0}$$

$$\overset{\hat{H}}{=} \lim_{x\to 0} \frac{\frac{\ln(1-x)}{x}(-1)}{\cos x}$$

$$= \lim_{x\to 0} -\frac{\ln(1-x)}{x\cos x} \qquad \text{I.F. } \frac{0}{0}$$

$$\overset{\hat{H}}{=} \lim_{x\to 0} \frac{-\frac{1}{1-x}\cdot(-1)}{\cos x + x(-\sin x)} = \frac{\frac{1}{1-0}}{\cos 0 + 0(-\sin 0)} = 1$$

$$= \lim_{b\to\infty} \int_2^b \frac{(\operatorname{arcsec} x)^p}{x\sqrt{x^2-1}}\,dx \qquad u = \operatorname{arcsec} x$$

$$du = \frac{1}{x\sqrt{x^2-1}}\,dx$$

$$= \lim_{b\to\infty} \int_{\pi/3}^{\operatorname{arcsec} b} u^p\,dx \qquad u(b) = \operatorname{arcsec} b$$

$$u(2) = \operatorname{arcsec} 2 = \pi/3$$

If $p=-1$ then

$$= \lim_{b\to\infty} \Big[\ln|u|\Big]_{\pi/3}^{\operatorname{arcsec} b} = \lim_{b\to\infty} \Big[\ln \underbrace{\operatorname{arcsec} b}_{\to\pi/2} - \ln \tfrac{\pi}{3}\Big]$$

$$= \ln \pi/2 - \ln \pi/3 = \ln 3/2$$

$\therefore$ converges when $p=-1$

If $p \neq -1$ then

$$= \lim_{b\to\infty} \left[\frac{u^{p+1}}{p+1}\right]_{\pi/3}^{\operatorname{arcsec} b} = \lim_{b\to\infty} \left[\frac{(\underbrace{\operatorname{arcsec} b}_{\to\pi/2})^{p+1}}{p+1} - \frac{(\pi/3)^{p+1}}{p+1}\right]$$

$$= \frac{(\pi/2)^{p+1}}{p+1} - \frac{(\pi/3)^{p+1}}{p+1}$$

$\therefore$ converges when $p \neq -1$

$\therefore$ converges $\forall p \in \mathbb{R}$