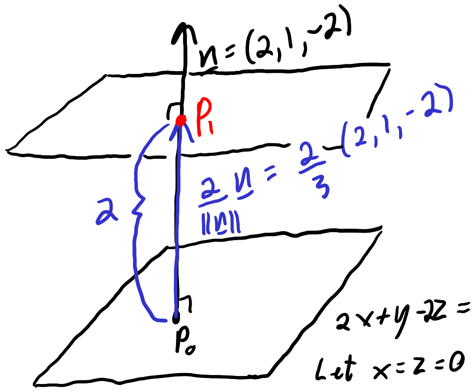
Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2023: **Quiz 10** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. If $\text{proj}_{\mathbf{a}}(\mathbf{v}) = \text{proj}_{\mathbf{a}}(\mathbf{u})$ then $\mathbf{v}$ might be equal to $\mathbf{u}$.

Question 2. *(5 marks)* Find the equation of a plane which is parallel to the plane $2x+y-2z=3$ and the distance between the two planes is 2.



$2x+y-2z=3$

Let $x=z=0$

$2(0)+y-2(0)=3$

$y=3$

$\therefore P_0(0,3,0)$ is on the given plane

$\vec{P_0P_1} = \frac{2}{3}(2,1,-2)$

$\vec{OP_1} - \vec{OP_0} = \frac{2}{3}(2,1,-2)$

$\vec{OP_1} = \vec{OP_0} + \frac{2}{3}(2,1,-2)$

$\vec{OP_1} = (0,3,0) + \left(\frac{4}{3}, \frac{2}{3}, \frac{-4}{3}\right)$

$\vec{OP_1} = \left(\frac{4}{3}, \frac{11}{3}, \frac{-4}{3}\right)$

$\therefore P_1\left(\frac{4}{3}, \frac{11}{3}, \frac{-4}{3}\right)$ lies on the plane that we are looking for.

$\therefore 2x+y-2z=d$

$2\left(\frac{4}{3}\right) + \frac{11}{3} - 2\left(\frac{-4}{3}\right) = d$

$\frac{27}{3} = d$

$\therefore 2x+y-2z=9$

note: there exists two such planes.

Question 3. *(4 marks)* Let $\mathbf{u}$, $\mathbf{v}$ be unit vectors in $\mathbb{R}^n$ and assume that they are all orthogonal to each other. Simplify completely: $\text{Proj}_{\mathbf{u}+\mathbf{v}}(\mathbf{u}-2\mathbf{v})$.

$$\text{proj}_{\underline{u}+\underline{v}}(\underline{u}-2\underline{v}) = \frac{(\underline{u}+\underline{v})\cdot(\underline{u}-2\underline{v})}{(\underline{u}+\underline{v})\cdot(\underline{u}+\underline{v})}(\underline{u}+\underline{v})$$

$$= \frac{\underline{u}\cdot\underline{u} + \underline{v}\cdot\underline{u} - 2\underline{v}\cdot\underline{u} - 2\underline{v}\cdot\underline{v}}{\underline{u}\cdot\underline{u} + \underline{u}\cdot\underline{v} + \underline{v}\cdot\underline{u} + \underline{v}\cdot\underline{v}}(\underline{u}+\underline{v})$$

$$= \frac{\|\underline{u}\|^2 + 0 - 2(0) - 2\|\underline{v}\|^2}{\|\underline{u}\|^2 + 0 + 0 + \|\underline{v}\|^2}(\underline{u}+\underline{v})$$

$$= \frac{(1)^2 - 2(1)}{(1)^2 + (1)^2}(\underline{u}+\underline{v})$$

$$= \frac{-1}{2}(\underline{u}+\underline{v})$$