**Dawson College**: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2023: **Quiz 11** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.** *(1 mark each)* Complete each of the following sentences with MUST, MIGHT, or CANNOT.

a. If $(1,1,0,0,0)$ and $(2,0,0,0,0)$ are both solutions of a system of 13 linear equations $A\mathbf{x}=\mathbf{b}$ then $(-1,1,0,0,0)$ must be a solution of the associated homogeneous system $A\mathbf{x}=\mathbf{0}$.

b. If the solution set of a system of 13 linear equations $A\mathbf{x}=\mathbf{b}$ is a line and $(1,1,0,0,0)$ and $(2,0,0,0,0)$ are both solutions of the system $\mathbf{x}=(-12,14,0,0,0)+t(-13,13,0,0,0)$, $t\in\mathbb{R}$ must is the solution set of $A\mathbf{x}=\mathbf{b}$.

c. If $(1,1,0,0,0)$ and $(2,0,0,0,0)$ are both solutions of a system of 13 linear equations $A\mathbf{x}=\mathbf{b}$ then $(-1,1,0,0,0)$ must be orthogonal to the rows of the coefficient matrix $A$.

d. If $(1,1,0,0,0)$ and $(2,0,0,0,0)$ are both solutions of a system of 13 linear equations $A\mathbf{x}=\mathbf{b}$ then $(0,1,0,0,0)$ might be orthogonal to the rows of the coefficient matrix $A$.

**Question 2.**

a. *(2 marks)* Find the parametric equation of the plane $x+2y+3z=5$. Find a point on the plane and two vectors parallel to the plane.

Let $y=s$, $z=t$, $s,t\in\mathbb{R}$

$x+2s+3t=5$

$x=5-2s-3t$

$\therefore (x,y,z)=(5-2s-3t,\ s,\ t)$

$=\underbrace{(5,0,0)}_{P_0}+s\underbrace{(-2,1,0)}_{d_1}+t\underbrace{(-3,0,1)}_{d_2}$

b. *(2 marks)* Find the intersection between the plane $x+2y+3z=5$ and the $xy$-plane.

The general equation of the xy-plane is $z=0$

$\left[\begin{array}{ccc|c}1&2&3&5\\0&0&1&0\end{array}\right] \sim_{-3R_2+R_1\to R_1} \left[\begin{array}{ccc|c}1&2&0&5\\0&0&1&0\end{array}\right]$

Let $y=t$

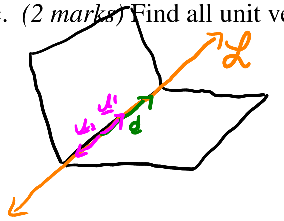
$x=5-2t$

$z=0$

$\therefore \mathscr{L}: (x,y,z)=(5-2t,\ t,\ 0)$

$=(5,0,0)+t\underbrace{(-2,1,0)}_{d}$

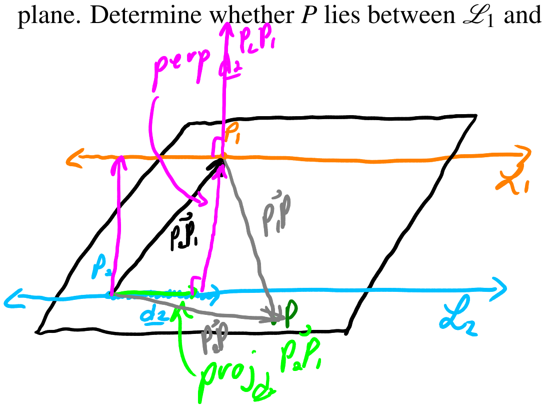
c. *(2 marks)* Find all unit vectors parallel to the plane $x+2y+3z=5$ and the $xy$-plane.




$u_1=\dfrac{d}{\|d\|}=\dfrac{(-2,1,0)}{\sqrt{5}}$

$u_2=\dfrac{-d}{\|d\|}=\dfrac{(2,-1,0)}{\sqrt{5}}$

**Question 3.** *(5 marks)* Given that $\mathscr{L}_1:\mathbf{x}=\overbrace{(1,0,2)}^{P_1}+t\overbrace{(-1,3,2)}^{d_1}$, $\mathscr{L}_2:\mathbf{x}=\underbrace{(1,1,-1)}_{P_2}+t\underbrace{(-1,3,2)}_{d_2}$ where $t\in\mathbb{R}$ and $P(0,5,-4)$ all lie on the same plane. Determine whether $P$ lies between $\mathscr{L}_1$ and $\mathscr{L}_2$.




$\vec{P_2P_1}=(1,0,2)-(1,1,-1)=(0,-1,3)$

$\vec{P_1P}=(0,5,-4)-(1,0,2)=(-1,5,-6)$

$\vec{P_2P}=(0,5,-4)-(1,1,-1)=(-1,4,-3)$

$\text{perp}_{d_2}\vec{P_2P_1}=(0,-1,3)-\dfrac{(0,-1,3)\cdot(-1,3,2)}{(-1,3,2)\cdot(-1,3,2)}(-1,3,2)$

$=(0,-1,3)-\dfrac{3}{14}(-1,3,2)$

$=\dfrac{1}{14}(3,-23,36)$

$\vec{P_1P}\cdot(3,-23,36)<0$

$\therefore$ angle between $\vec{P_1P}$ and $\text{perp}_{d_2}\vec{P_2P_1}$ is obtuse

$\vec{P_2P}\cdot(3,-23,26)<0$

$\therefore$ angle between $\vec{P_1P}$ and $\text{perp}_{d_2}\vec{P_2P_1}$ is obtuse

$\therefore$ $P$ does not lie between $\mathscr{L}_1$ and $\mathscr{L}_2$.

**Bonus Question.** *(3 marks)* Given a *Yann plane segment* defined as $\mathbf{x}=(1,0,2)+s(1,1,1)+t(2,1,3)$ where $(s,t)\in[-1,2]\times[-2,0]$. Find the area of the Yann plane segment.