Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2023: **Quiz 12** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Consider the vectors in $\mathbb{R}^3$: $\mathbf{u}(\theta) = (\cos\theta, \sin\theta, 0)$ and $\mathbf{v} = (1, 0, 1)$. Find all the values of the angle θ in $[0, 2\pi)$ for which the parallelepiped spanned by $\mathbf{u}(\theta)$, $\mathbf{v}$ and $\mathbf{u}(\theta) \times \mathbf{v}$ has volume $V = 2$.

$$\underline{u}(\theta) \times \underline{v} = \left(\begin{vmatrix} \sin\theta & 0 \\ 0 & 1 \end{vmatrix}, - \begin{vmatrix} \cos\theta & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} \cos\theta & 1 \\ \sin\theta & 0 \end{vmatrix} \right) = (\sin\theta, -\cos\theta, -\sin\theta)$$

$$\begin{matrix} \cos\theta & 1 \\ \sin\theta & 0 \\ 0 & 1 \end{matrix}$$

$$V = |\underline{u}(\theta) \cdot (\underline{v} \times (\underline{u}(\theta) \times \underline{v}))|$$

$$V = |(\underline{u}(\theta) \times \underline{v}) \cdot (\underline{u}(\theta) \times \underline{v})|$$

$$V = \|\underline{u}(\theta) \times \underline{v}\|^2$$

$$2 = \sin^2\theta + \cos^2\theta + \sin^2\theta$$

$$2 = 1 + \sin^2\theta$$

$$1 = \sin^2\theta$$

$$\pm 1 = \sin\theta$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

Question 2. *(5 marks)* Given the lines $L_1 : \begin{cases} x = 7+2s \\ y = 1 \\ z = 6+s \end{cases}$ and $L_2 : \begin{cases} x = 5-t \\ y = -1-t \\ z = -6+t \end{cases}$, find the parametric equations of the line that intersects both L_1 and L_2 at right angles.

$\underline{d}_1 = (2,0,1)$, $\underline{d}_2 = (-1,-1,1)$ since $\underline{d}_1 \nparallel \underline{d}_2$ the lines are not parallel.

Lets determine if the lines intersect

① $7+2s = 5-t$

$1 = -1-t \Rightarrow t = -2$

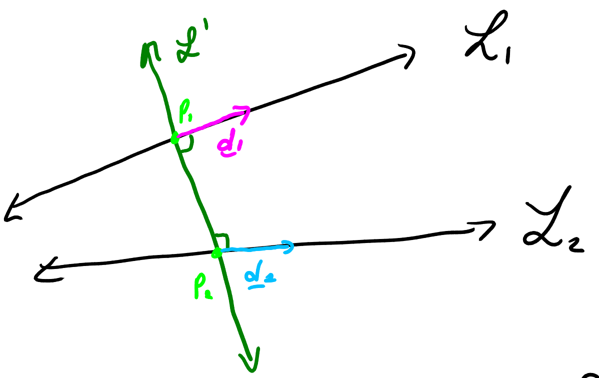
③ $6+s = -6+t$

sub $t=-2$ in ① $7+2s = 5-(-2) \Rightarrow s = 0$

sub $t=-2$ in ③ $6+s = -6+(-2) \Rightarrow s = -14$

$\therefore \nexists\, s,t.$ such that the two lines have a point in common.

$\therefore \mathcal{L}_1$ and $\mathcal{L}_2$ are skew lines.



$\mathcal{L}'$ the line which passes through $\mathcal{L}_1$ and $\mathcal{L}_2$ at $\perp$ is the line which passes through the the closest points on $\mathcal{L}_1$ and $\mathcal{L}_2$.

$\vec{P_1P_2} = (5-t, -1-t, -6+t) - (7+2s, 1, 6+s)$

$= (-2-t-2s, -2-t, -12+t-s)$

① $\vec{P_1P_2} \cdot \underline{d}_1 = 0$ ①' $(-2-t-2s, -2-t, -12+t-s) \cdot (2,0,1) = 0$

② $\vec{P_1P_2} \cdot \underline{d}_2 = 0$ ②' $($ " , " , " $) \cdot (-1,-1,1) = 0$

①'' $2(-2-t-2s) + (-12+t-s) = 0$

②'' $-1(-2-t-2s) - (-2-t) + (-12+t-s) = 0$

①''' $-16-t-5s = 0$

②''' $-8+3t+s = 0$

②''' $+3$①''' : $-56-14s = 0 \Rightarrow s = -4 \Rightarrow t = 4$

$\Rightarrow \underline{d}' = (-2-(4)-2(-4), -2-(4), -12+(4)-(-4))$

$= (2,-6,-4)$

and $P_1: (x,y,z) = (7+2(-4), 1, 6+(-4)) = (-1,1,2)$

$\therefore \mathcal{L}' : (x,y,z) = (-1,1,2) + t(2,-6,-4) \quad t \in \mathbb{R}$