**Dawson College**: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2023: **Quiz 13** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.**[1] Let $V = \{(a,b) \mid a,b \in \mathbb{R}, b > 0\}$. And the addition in $V$ is defined by $(a,b) \bigoplus (c,d) = (ad+bc, bd)$ and scalar multiplication in $V$ is defined by $t \bigodot (a,b) = (tab^{t-1}, b^t)$

a. *(1 mark)* $(4,2) \bigoplus (-5,1)$ $= (4(1)+2(-5), 2(1)) = (-6,2)$

b. *(1 mark)* $-2 \bigodot (1,2)$ $= (-2(1)(2)^{-2-1}, 2^{-2}) = (\frac{-2}{2^3}, \frac{1}{2^2}) = (\frac{-1}{4}, \frac{1}{4})$

c. *(3 marks)* Demonstrate whether the 4th axiom of vector spaces holds. That is, does the zero vector exists in $V$.

Let $\underline{0} = (a,b)$ and $\underline{v} = (v_1, v_2)$

$\underline{v} + \underline{0} = \underline{v}$

$(av_2 + bv_1, bv_2) = (v_1, v_2)$

$\Rightarrow av_2 + bv_1 = v_1$ and $bv_2 = v_2$

$b = 1$

$av_2 + 1v_1 = v_1$

$av_2 = 0$

$\Rightarrow a = 0$

$\therefore \underline{0} = (0,1) \in V$ since $1 > 0$ and $0 \in \mathbb{R}$

d. *(3 marks)* Demonstrate whether the 5th axiom of vector spaces holds. That is, additive inverses in $V$ exists for all vectors in $V$.

Let $\underline{v} = (v_1, v_2) \in V$ and $\underline{w} = (w_1, w_2)$

$\underline{v} + \underline{w} = \underline{0}$

$(v_1, v_2) + (w_1, w_2) = (0,1)$

$(v_1 w_2 + v_2 w_1, v_2 w_2) = (0,1)$

$\Rightarrow v_1 w_2 + v_2 w_1 = 0$ $\quad v_2 w_2 = 1$

$w_2 = \frac{1}{v_2} > 0$ since $v_2 > 0$

$\Rightarrow v_1 \frac{1}{v_2} + v_2 w_1 = 0$

$w_1 = \frac{-v_1}{v_2^2} \in \mathbb{R}$ since $v_i \in \mathbb{R}$

$\therefore \underline{w} \in V$.

**Question 2.** *(5 marks)* Prove: If $a \neq 1$ and $\mathbf{v}$ is any vector in a vector space $V$ such that $a\mathbf{v} = \mathbf{v}$ then $\mathbf{v} = \mathbf{0}$. *Show every step, justify every step, and cite the axiom(s) used!!!*

Suppose $\underline{v} \neq \underline{0}$

$a\underline{v} = \underline{v}$

$a\underline{v} + \underline{w} = \underline{v} + \underline{w}$ add the additive inverse of $\underline{v}$, existance by axiom 5

$a\underline{v} + \underline{w} = \underline{0}$ axiom 5

$a\underline{v} + (-1)\underline{v} = \underline{0}$ by thm 1.1 seen in class

$(a-1)\underline{v} = \underline{0}$ axiom 7

By thm 1.1 $\underline{v} = 0$ or $a - 1 = 0$ but since $\underline{v} \neq \underline{0}$ then $a - 1 = 0$ $\quad a = 1$ ↯ $\therefore \underline{v} = \underline{0}$

**Bonus Question.** *(3 mark)* Show that the commutativity axiom can be shown using the other axioms by first calculating $(1+1)(\mathbf{v}+\mathbf{w})$ in two different ways.

[1] From http://www.math.uwaterloo.ca/ jmckinno/Math225/Week1/Lecture1e.pdf