

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531***. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. (4 marks) Determine whether $H = \{A \mid A \in \mathcal{M}_{n \times n} \text{ and } BAC = CAB\}$ where B and C are fixed $n \times n$ matrices, is a subspace of $\mathcal{M}_{n \times n}$

Question 2. (4 marks) Let $\{A_1, A_2, \dots, A_k\}$ be linearly independent in $\mathcal{M}_{m \times n}$, and suppose that U and V are invertible matrices of size $m \times m$ and $n \times n$, respectively. Show that $\{UA_1V, UA_2V, \dots, UA_kV\}$ is linearly independent.

Question 3. (4 marks) Determine whether the following statements is true or false. If the statement is false, provide a counterexample. If the statement is true, provide a proof of the statement.

If $\mathbf{u}, \mathbf{v}, \mathbf{p}, \mathbf{q}$ be nonzero vectors in \mathbb{R}^3 and $\text{span}\{\mathbf{u}, \mathbf{v}\} = \text{span}\{\mathbf{p}, \mathbf{q}\}$ then $(\mathbf{u} \times \mathbf{v}) \times (\mathbf{p} \times \mathbf{q}) = \mathbf{0}$.