Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2023: **Quiz 15** name: ____________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(2 marks)* Consider the vectors $\mathbf{u} = (1, 2, -3)$ and $\mathbf{v} = (0, 7, 1)$ where $\mathscr{B} = \{\mathbf{u}, \mathbf{v}\}$ is a basis for a vector space W. Find the coordinates of the vector $\mathbf{w} = (1, 23, 0)$ relative to $\mathscr{B}$.

Question 2. *(5 marks)* Consider the space $M_{3\times 3}$ of all square matrices of size 3×3 and the subset W of skew-symmetric matrices:

$$W = \left\{ \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix},\ a, b, c \text{ in } \mathbb{R} \right\}$$

and consider the matrices:

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

Determine whether $S = \{A, B, C\}$ is a basis for W and state the dimension of W.

Question 3. *(5 marks)* Let A be a nonzero 2×2 matrix and write $U = \{X \in \mathscr{M}_{2\times2} \mid XA = AX\}$. Show that $\dim(U) \geq 2$. *Hint: I and A are in U and prove by contradiction.*

Bonus Question.
definition:[1] A *group* G is a set of elements satisfying the four conditions below, relative to some binary operation.

1. $\exists e \in G \, \forall g \in G : e \cdot g = g \cdot e = g$. (Identity.)
2. $\forall x,y,z \in G : (x \cdot y) \cdot z = x \cdot (y \cdot z)$. (Associativity.)
3. $\forall x \in G, \exists y \in G \, x \cdot y = y \cdot x = e$. (Inverse.)
4. $\forall x,y \in G : x \cdot y \in G$. (Closure.)

where e is called the *identity*.

a. *(2 marks)* Show that the identity is unique.
b. *(2 marks)* Is a vector space a group?

[1]https://web.williams.edu/Mathematics/sjmiller/public_html/mathlab/public_html/handouts/GroupTheoryIntro.tex