Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2023: **Quiz 1** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If the number of equations in a linear system is strickly less than the number of unknowns, then the system must be inconsistent.

False, $x+y=0$ has infinitely many solutions. Namely $\{(-t,t) \mid t \in \mathbb{R}\}$

Question 2. *(3 marks)* In each of the following, find (if possible) conditions on a and b such that the system has no solution, one solution, and infinitely many solutions.

$$\begin{cases} ax + y = 1 \\ 2x + y = b \end{cases} \Rightarrow \begin{cases} y = -ax + 1 & (1) \\ y = -2x + b & (2) \end{cases}$$

No Solutions:
Two lines will have no points in common if they are parallel and not identical. That is, if $a=2$ and $b \neq 1$

One Solution:
Two lines will have a unique point in common if they have different slopes. $\therefore$ unique solution if $a \neq 2$

∞ many solutions:
Two lines will have infinitely many points in common if they are identical. $\therefore a=2$ and $b=1$

Question 3. *(2 marks)* Multiplying a row by zero is not an elementary row operation because it does not neccessarily preserve the solution set. Find an example where it does preserve the solution set. Find an example where it increases the number of solutions.

Preserve the solution set:

(1) $\begin{cases} x=1 \\ 2x=2 \end{cases}$ $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ $\xrightarrow{0R_2 \to R_2}$ $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

(2) $\{ x=1$

Both (1) and (2) have the same solution set.

Increases the solution set:

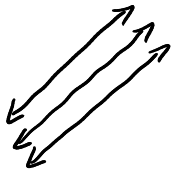
(1)' $\begin{cases} x+y=0 \\ x-y=0 \end{cases}$ has $(0,0)$ as a solution set

$\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \end{bmatrix}$ $\xrightarrow{0R_2 \to R_2}$ $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

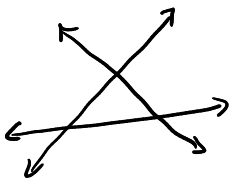
(2)' $\{x+y=0$ has ∞ as a solution set

Question 4. *(2 marks)* Illustrate **all** relative positions of lines in an inconsistent linear system consisting of three lines.

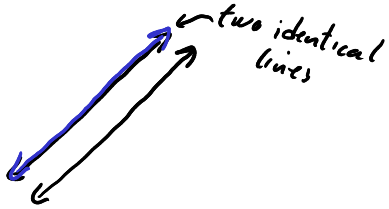
Case 1



Case 2



Case 3



Case 4