Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2023: **Quiz 2** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Find the value(s) of a, if any, for which the following system

$$\begin{cases} x + y + 3z = 2 \\ -3x + (a-1)y - 10z = -7 \\ 4x + (2-a)y + (a^2-8)z = a-2 \end{cases}$$

has

a. exactly one solutions,
b. infinitely many solutions,
c. no solutions.

$$\begin{bmatrix} 1 & 1 & 3 & 2 \\ -3 & a-1 & -10 & -7 \\ 4 & 2-a & a^2-8 & a-2 \end{bmatrix} \sim \begin{matrix} 3R_1+R_2\to R_2 \\ -4R_1+R_3\to R_3 \end{matrix} \begin{bmatrix} 1 & 1 & 3 & 2 \\ 0 & a+2 & -1 & -1 \\ 0 & -2-a & a^2-20 & a-10 \end{bmatrix}$$

$$\sim R_2+R_3\to R_3 \begin{bmatrix} 1 & 1 & 3 & 2 \\ 0 & a+2 & -1 & -1 \\ 0 & 0 & a^2-21 & a-11 \end{bmatrix}$$

a) #leading entries in var. col. = #var.

$a+2\neq 0 \Rightarrow a\neq -2$

$a^2-21\neq 0 \Rightarrow a\neq \pm\sqrt{21}$ $\quad\therefore a\in\mathbb{R}\setminus\{-\sqrt{21},-2,\sqrt{21}\}$

c) If $a=-2$ then $\begin{bmatrix} 1 & 1 & 3 & 2 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -17 & -13 \end{bmatrix} \sim -17R_2+R_3 \begin{bmatrix} 1 & 1 & 3 & 2 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ $\therefore$ no solution when $a=-2$ since there is a leading entry in the constant col.

If $a=\pm\sqrt{21}$ $\begin{bmatrix} 1 & 1 & 3 & 2 \\ 0 & \pm\sqrt{21}+2 & -1 & -1 \\ 0 & 0 & 0 & \pm\sqrt{21}-11 \end{bmatrix}$ $\therefore$ no solution since there is a leading entry in the constant column.

b) No values of 'a' when #leading entries in the var. col. < #var. For any value of a the system will not have ∞ many solutions.

Question 2. *(5 marks)* Prove that if $ad-bc\neq 0$, then the reduced row echelon form of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Lets perform Gauss-Jordan on $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Case $a\neq 0$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \sim \underset{\text{since } a\neq 0}{aR_2\to R_2} \begin{bmatrix} a & b \\ ac & ad \end{bmatrix} \sim -cR_1+R_2\to R_2 \begin{bmatrix} a & b \\ 0 & ad-bc \end{bmatrix} \sim \underset{\text{since } ad-bc\neq 0}{\frac{1}{ad-bc}R_2\to R_2} \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$

$\sim -bR_2+R_1\to R_1 \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix}$

$\sim \frac{1}{a}R_1\to R_1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ since $a\neq 0$

Case $a=0$

Then $ad-bc\neq 0$

$0(d)-bc\neq 0$

$-bc\neq 0 \Rightarrow b\neq 0$ and $c\neq c$

$\begin{bmatrix} 0 & b \\ c & d \end{bmatrix} \sim R_1\leftrightarrow R_2 \begin{bmatrix} c & d \\ 0 & b \end{bmatrix} \sim \frac{1}{b}R_2\to R_2 \begin{bmatrix} c & d \\ 0 & 1 \end{bmatrix}$ since $b\neq 0$

$\sim -dR_2+R_1\to R_1 \begin{bmatrix} c & 0 \\ 0 & 1 \end{bmatrix} \sim \frac{1}{c}R_1\to R_1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ since $c\neq 0$