**Dawson College**: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2023: **Quiz 3** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.** *(4 marks)* Solve for $A$, if possible.

$$\begin{bmatrix}3 & 1\\1 & -1\end{bmatrix}A - A\begin{bmatrix}2 & 1\\0 & 1\end{bmatrix} = \begin{bmatrix}9 & -21\\-23 & -7\end{bmatrix}$$

$A = \begin{bmatrix}a & b\\c & d\end{bmatrix}$

$\begin{bmatrix}3 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}a & b\\c & d\end{bmatrix} - \begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}2 & 1\\0 & 1\end{bmatrix} = \begin{bmatrix}9 & -21\\-23 & -7\end{bmatrix}$

$\begin{bmatrix}3a+c & 3b+d\\a-c & b-d\end{bmatrix} - \begin{bmatrix}2a & a+b\\2c & c+d\end{bmatrix} = \begin{bmatrix}9 & -21\\-23 & -7\end{bmatrix}$

$\begin{bmatrix}a+c & d+2b-a\\a-3c & b-c-2d\end{bmatrix} = \begin{bmatrix}9 & -21\\-23 & -7\end{bmatrix}$

(1) $a+c = 9$
(2) $a-3c = -23$
(3) $-a+2b+d = -21$
(4) $b-c-2d = -7$

(1) - (2) : $4c = 32$
$c = 8$ sub into (1) $a+8 = 9$
$a = 1$

sub $a$ and $c$ into (3) and (4)

(3)' $2b+d = -20$
(4)' $b-2d = 1$

(3)' - 2(4)' : $5d = -22$
$d = -\frac{22}{5}$ sub into (4)' $b - 2\left(-\frac{22}{5}\right) = 1$
$b = -\frac{39}{5}$

$\therefore A = \begin{bmatrix}1 & -39/5\\8 & -22/5\end{bmatrix}$

**Question 2.** *(4 marks)* A matrix $B$ is said to be a *square root* of a matrix $A$ if $BB = A$. Find all square roots of $\begin{bmatrix}5 & 0\\0 & 9\end{bmatrix}$.

Let $B = \begin{bmatrix}a & b\\c & d\end{bmatrix}$ then $BB = A$

$\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}a & b\\c & d\end{bmatrix} = \begin{bmatrix}5 & 0\\0 & 9\end{bmatrix}$

$\begin{bmatrix}a^2+bc & ab+bd\\ac+dc & bc+d^2\end{bmatrix} = \begin{bmatrix}5 & 0\\0 & 9\end{bmatrix}$

(1) $a^2+bc = 5$
(2) $ab+bd = 0 \Rightarrow b(a+d) = 0$
(3) $ac+dc = 0$
(4) $bc+d^2 = 9$

To satisfy eqn (2) either $b=0$ or $a+d=0$

If $b=0$ then (1) and (4) simplify to
$a^2 = 5 \Rightarrow a = \pm\sqrt{5}$
$d^2 = 9 \Rightarrow d = \pm 3$

in order to satisfy (3) $c = 0$

$\therefore$ 4 solutions $B = \begin{bmatrix}\pm\sqrt{5} & 0\\0 & \pm 3\end{bmatrix}$

If $a+d=0$
$a = -d$
sub into (1) : $(-d)^2 + bc = 5$
$d^2 + bc = 5$
and (4) : $d^2 + bc = 9$
Impossible to satisfy (1) and (4)
$\therefore$ only the 4 solutions listed above.

**Question 3.** *(4 marks)* Prove: That the trace is a linear operator. That is, if $A$ and $B$ are $n \times n$ matrices and $\alpha, \beta$ are scalars then $\operatorname{trace}(\alpha A + \beta B) = \alpha \operatorname{trace}(A) + \beta \operatorname{trace}(B)$.

$\operatorname{trace}(\alpha A + \beta B) = \operatorname{trace}(\alpha[a_{ij}] + \beta[b_{ij}])$
$= \operatorname{trace}([\alpha a_{ij}] + [\beta b_{ij}])$
$= \operatorname{trace}([\alpha a_{ij} + \beta b_{ij}])$
$= (\alpha a_{11} + \beta b_{11}) + (\alpha a_{22} + \beta b_{22}) + \dots + (\alpha a_{nn} + \beta b_{nn})$
$= \alpha a_{11} + \alpha a_{22} + \dots + \alpha a_{nn} + \beta b_{11} + \beta b_{22} + \dots + \beta b_{nn}$
$= \alpha(a_{11} + a_{22} + \dots + a_{nn}) + \beta(b_{11} + b_{22} + \dots + b_{nn})$
$= \alpha \operatorname{trace}(A) + \beta \operatorname{trace}(B)$