Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S1: Winter 2023: **Quiz 3** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(4 marks)* Solve for A, if possible.

$$\begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix} A - A \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & -21 \\ -23 & -7 \end{bmatrix}$$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & -21 \\ -23 & -7 \end{bmatrix}$

$\begin{bmatrix} 3a+c & 3b+d \\ a-c & b-d \end{bmatrix} - \begin{bmatrix} 2a & a+b \\ 2c & c+d \end{bmatrix} = \begin{bmatrix} 9 & -21 \\ -23 & -7 \end{bmatrix}$

$\begin{bmatrix} a+c & d+2b-a \\ a-3c & b-c-2d \end{bmatrix} = \begin{bmatrix} 9 & -21 \\ -23 & -7 \end{bmatrix}$

① $a+c=9$
② $a-3c=-23$
③ $-a+2b+d=-21$
④ $b-c-2d=-7$

① - ② : $4c = 32$
$c = 8$ sub into ① $a+8=9$
$a=1$

sub a and c into ③ and ④
③' $2b+d=-20$
④' $b-2d=1$

③' - 2④' : $5d=-22$
$d=-\frac{22}{5}$ sub into ④' $b-2\left(\frac{-22}{5}\right)=1$
$b=\frac{-39}{5}$

$\therefore A = \begin{bmatrix} 1 & -39/5 \\ 8 & -22/5 \end{bmatrix}$

Question 2. *(4 marks)* A matrix B is said to be a *square root* of a matrix A if $BB = A$. Find all square roots of $\begin{bmatrix} 5 & 0 \\ 0 & 9 \end{bmatrix}$.

Let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $BB=A$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 9 \end{bmatrix}$

$\begin{bmatrix} a^2+bc & ab+bd \\ ac+dc & bc+d^2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 9 \end{bmatrix}$

① $a^2+bc=5$
② $ab+bd=0 \Rightarrow b(a+d)=0$
③ $ac+dc=0$
④ $bc+d^2=9$

To satisfy eqn ② either $b=0$ or $a+d=0$

If $b=0$ then ① and ④ simplify to
$a^2=5 \Rightarrow a=\pm\sqrt{5}$
$d^2=9 \Rightarrow d=\pm 3$

in order to satisfy ③ $c=0$

$\therefore$ 4 solutions $B = \begin{bmatrix} \pm\sqrt{5} & 0 \\ 0 & \pm 3 \end{bmatrix}$

If $a+d=0$
$a=-d$
sub into ① : $(-d)^2+bc=5$
$d^2+bc=5$
and ④ : $d^2+bc=9$
Impossible to satisfy ① and ④
$\therefore$ Only the 4 solutions listed above.

Question 3. *(4 marks)* Prove: That the trace is a linear operator. That is, if A and B are $n \times n$ matrices and α, β are scalars then $\operatorname{trace}(\alpha A + \beta B) = \alpha \operatorname{trace}(A) + \beta \operatorname{trace}(B)$.

$\operatorname{trace}(\alpha A + \beta B) = \operatorname{trace}(\alpha[a_{ij}] + \beta[b_{ij}])$
$= \operatorname{trace}([\alpha a_{ij}] + [\beta b_{ij}])$
$= \operatorname{trace}([\alpha a_{ij} + \beta b_{ij}])$
$= (\alpha a_{11} + \beta b_{11}) + (\alpha a_{22} + \beta b_{22}) + \cdots + (\alpha a_{nn} + \beta b_{nn})$
$= \alpha a_{11} + \alpha a_{22} + \cdots + \alpha a_{nn} + \beta b_{11} + \beta b_{22} + \cdots + \beta b_{nn}$
$= \alpha(a_{11} + a_{22} + \cdots + a_{nn}) + \beta(b_{11} + b_{22} + \cdots + b_{nn})$
$= \alpha \operatorname{trace}(A) + \beta \operatorname{trace}(B)$